# Exposing the plural nature of molecular clouds

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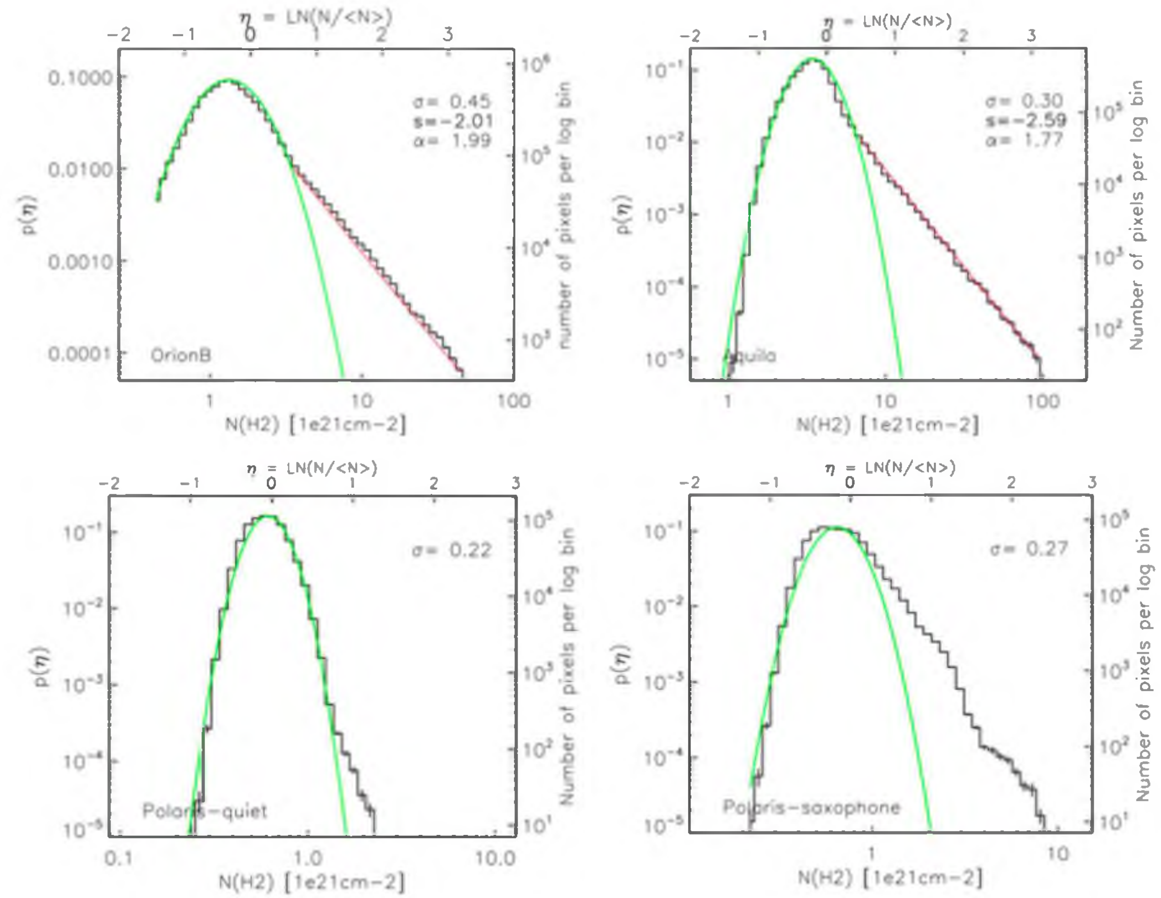
# Statistical measurements on molecular clouds

- Probability distribution function (PDF) : the foundation of many modern theories of star formation
    - Because the density distribution of the ISM is very complex, it is dangerous to mix different evolutionary stages over a same region (Hill et al. 2011).
- Power spectrum analysis : one of the best tools to analyse and characterise the turbulence
    - If molecular clouds are dominated by turbulence, the power spectrum should be a complete measure of the density distribution (Green 93, Stutzki et al. 98)

Schneider et al. 2013




Miville-Deschênes et al. 2010

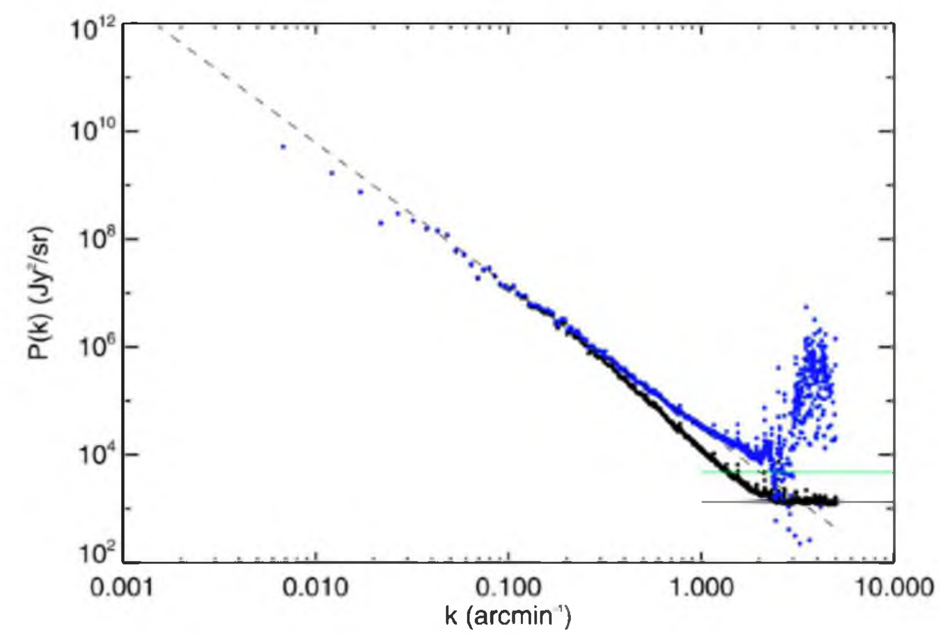


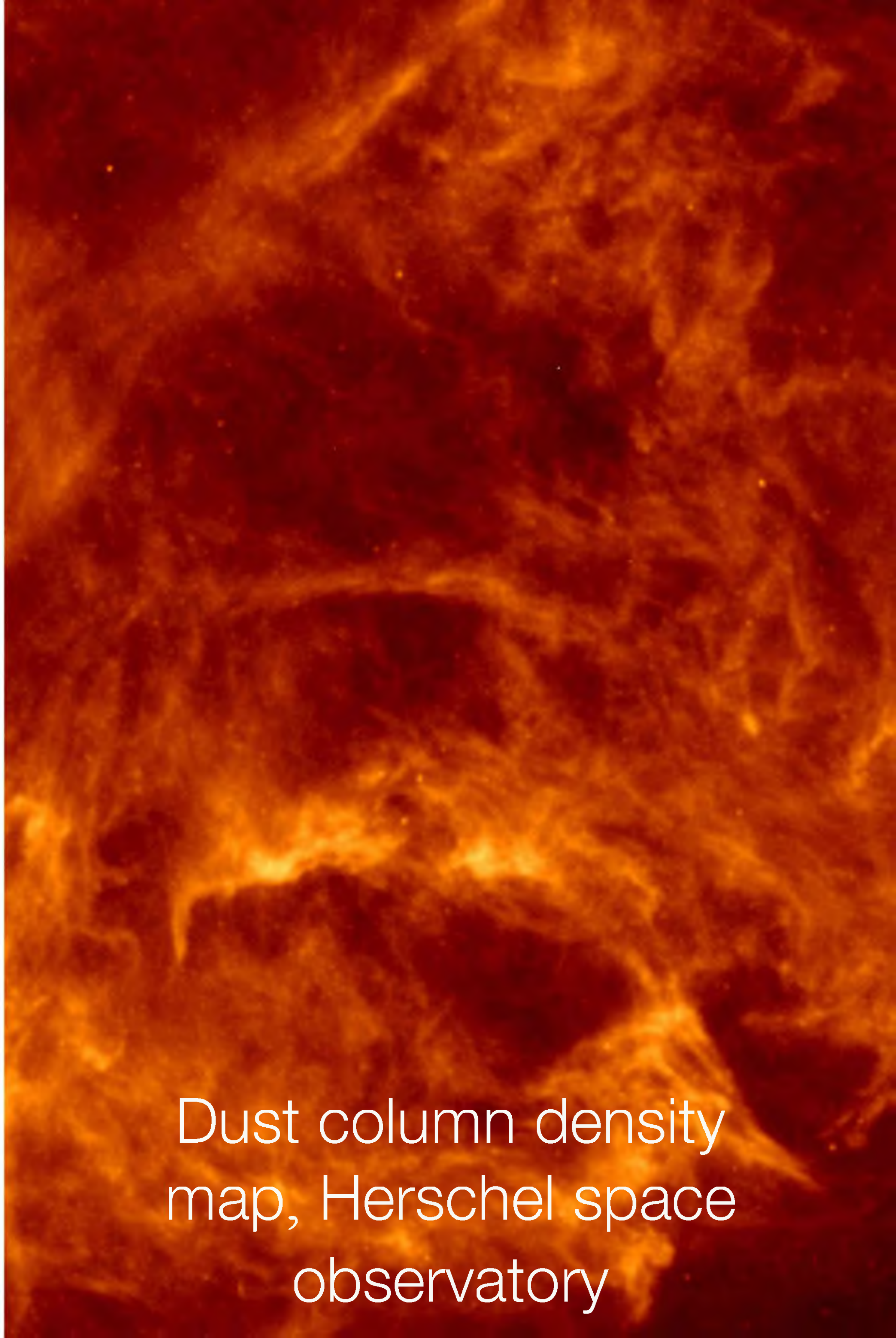

**Fig. 3.** Power spectrum of the SPIRE 250 $\mu$m map of the Polaris flare.

# Statistical measurements on molecular clouds

- Power spectrum analysis:

  - Panopoulou et al. 2017 -> search of a characteristic scale in the presence of a characteristic width.

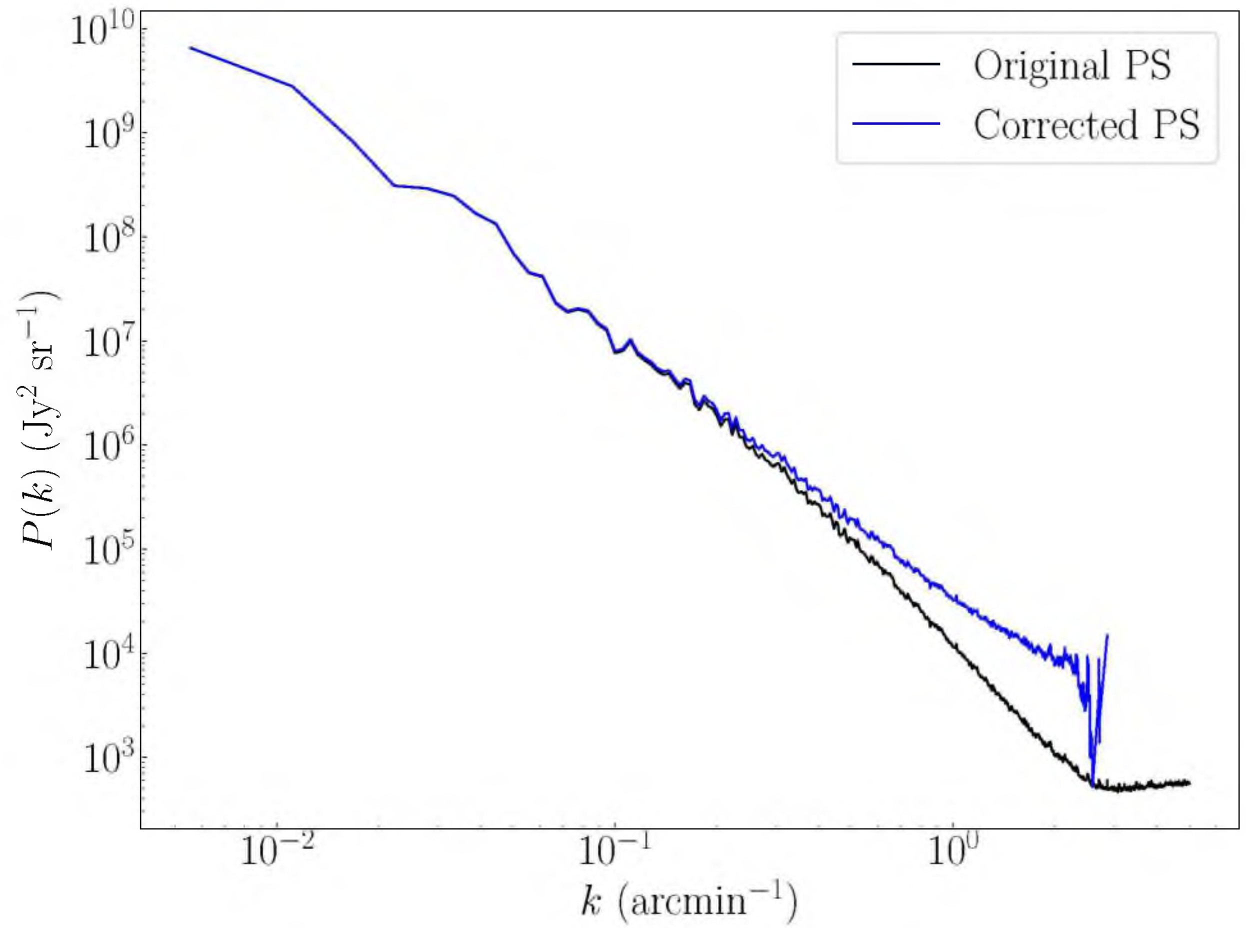
  - Roy et al. 2019 suggest that only filamentary structures with a high area filling factor and/or a high column-density contrast have an impact on the scale-free power spectrum of dust continuum images.



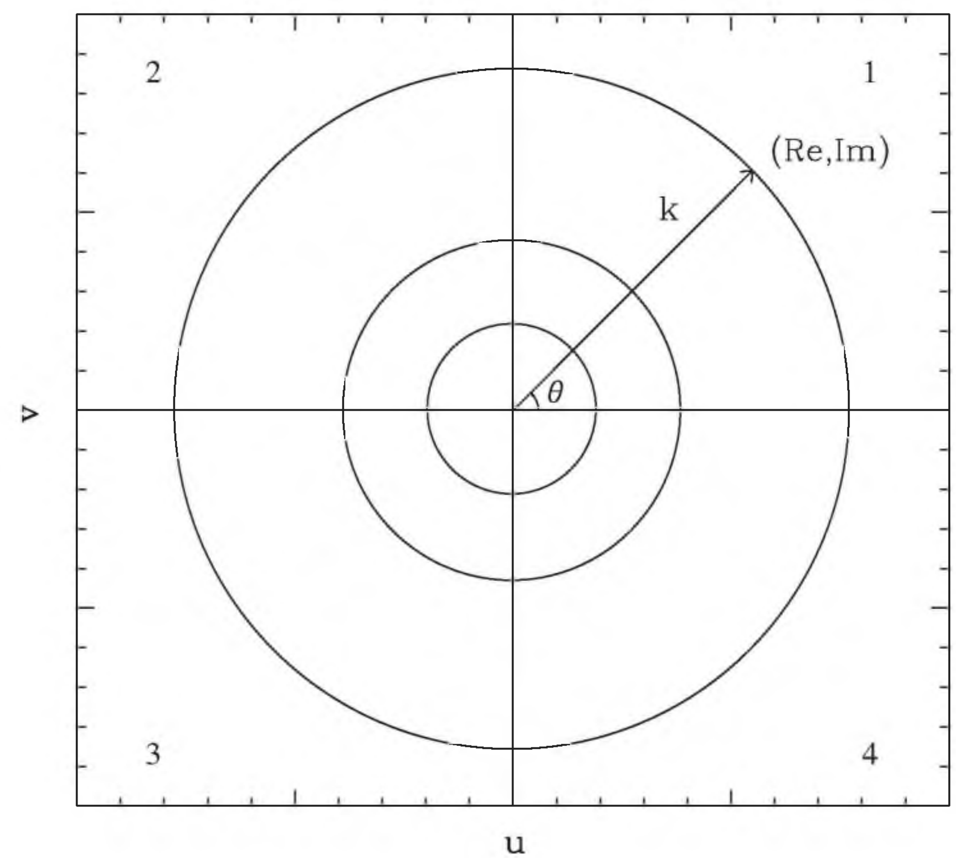
# Power spectrum analysis

# Power spectrum analysis

GBHS Polaris 250 μm map




Fourier transform




Power

$$P(k) = \langle |A(k)|^2 \rangle_\theta$$

Amplitude & Phase

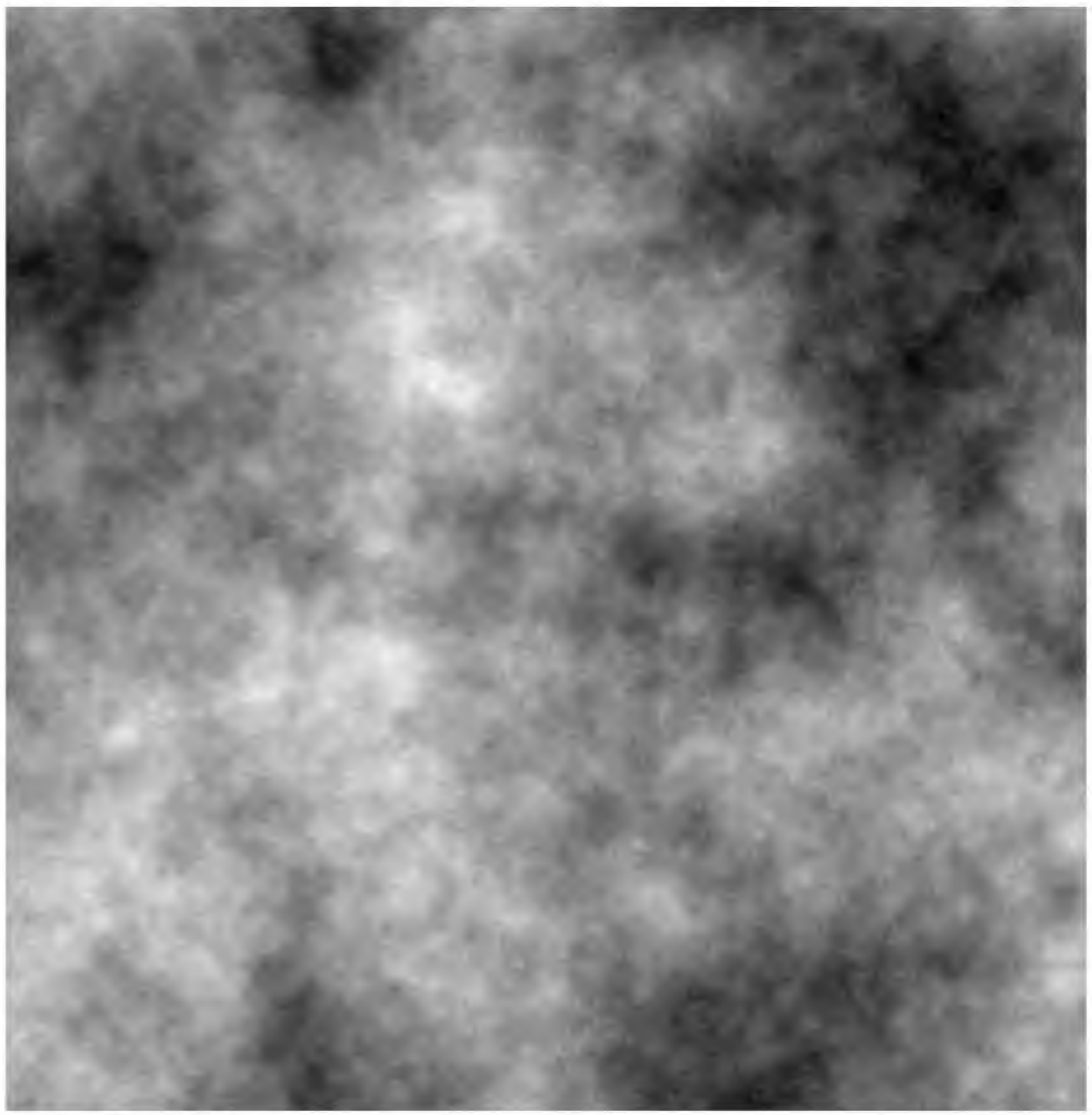
$$A = \sqrt{Re^2 + Im^2}$$

$$\phi = \tan^{-1}(Im/Re)$$

Wavenumber

$$k = \sqrt{u^2 + v^2}$$

# Fractional Brownian motion (fBm)
# Self-similar distribution



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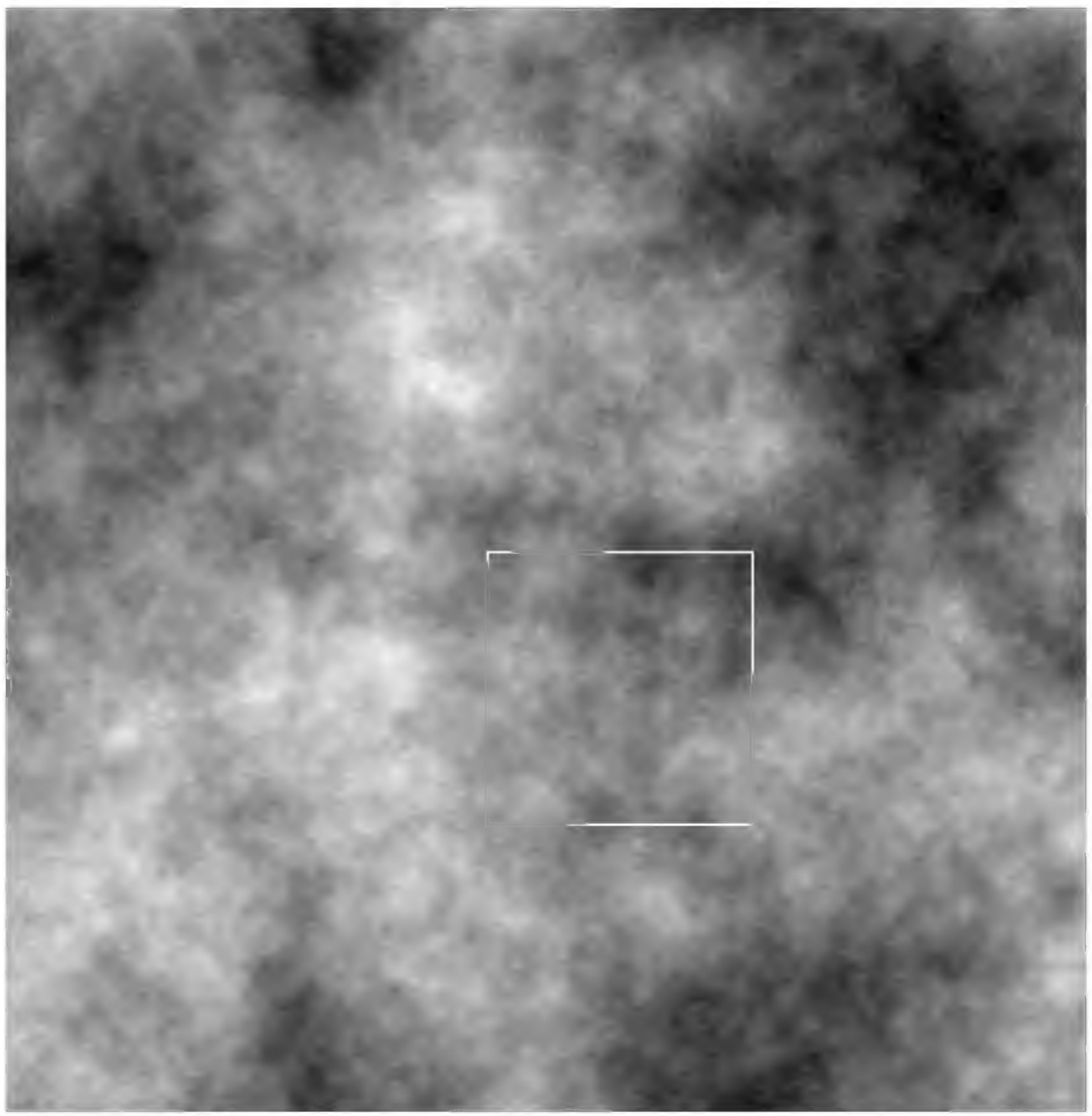

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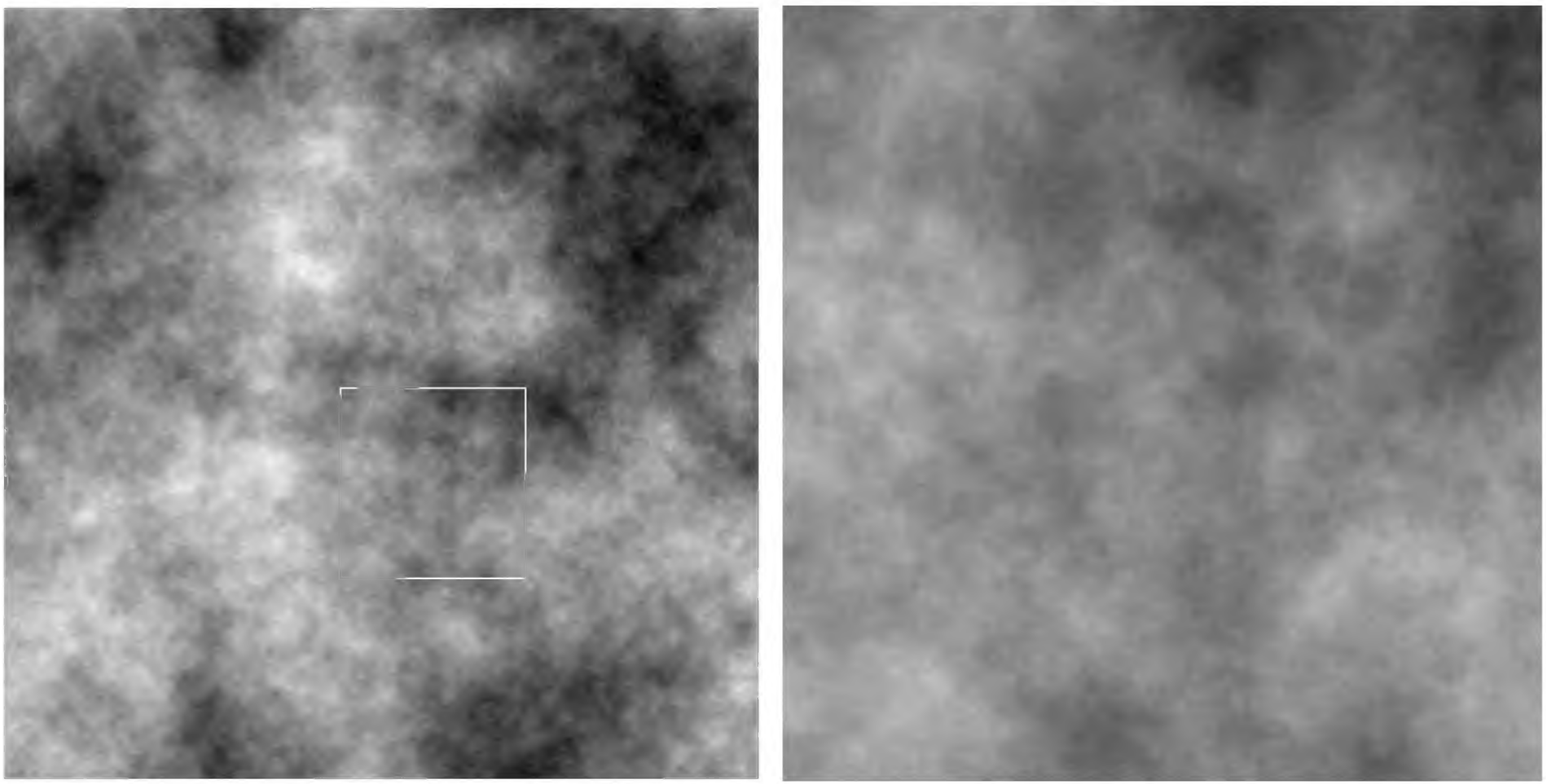

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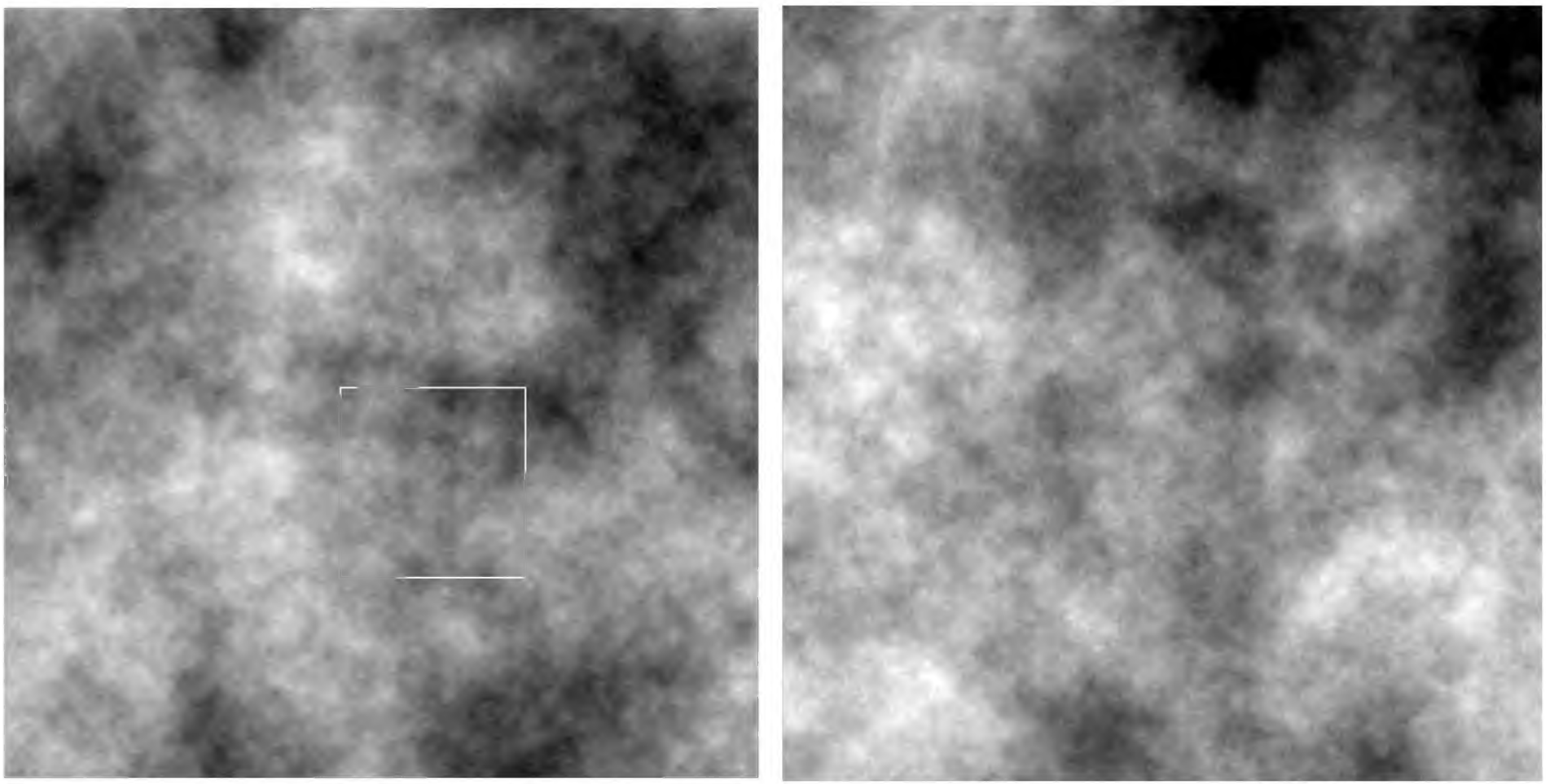

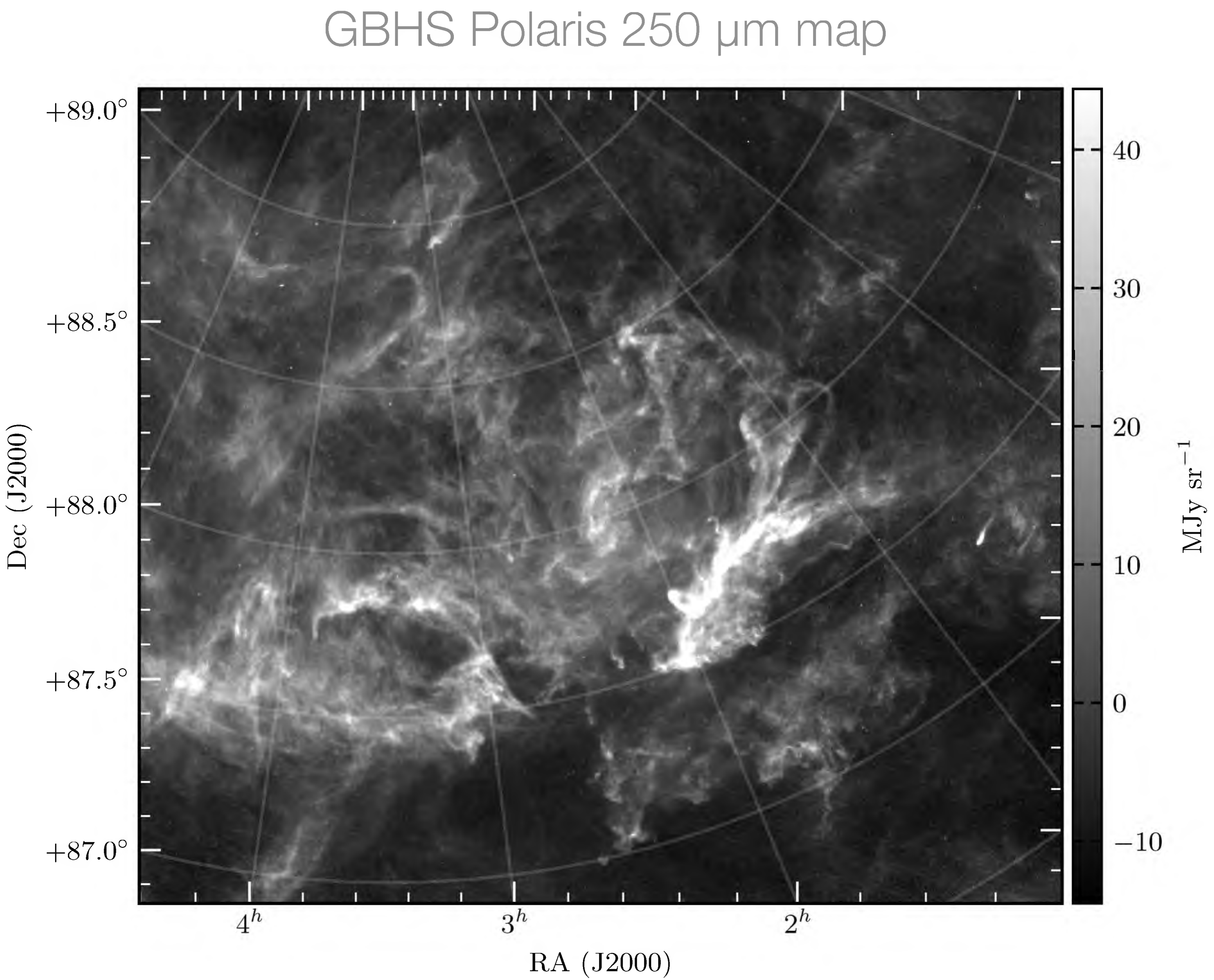
GBHS Polaris 250 μm map
+89.0°
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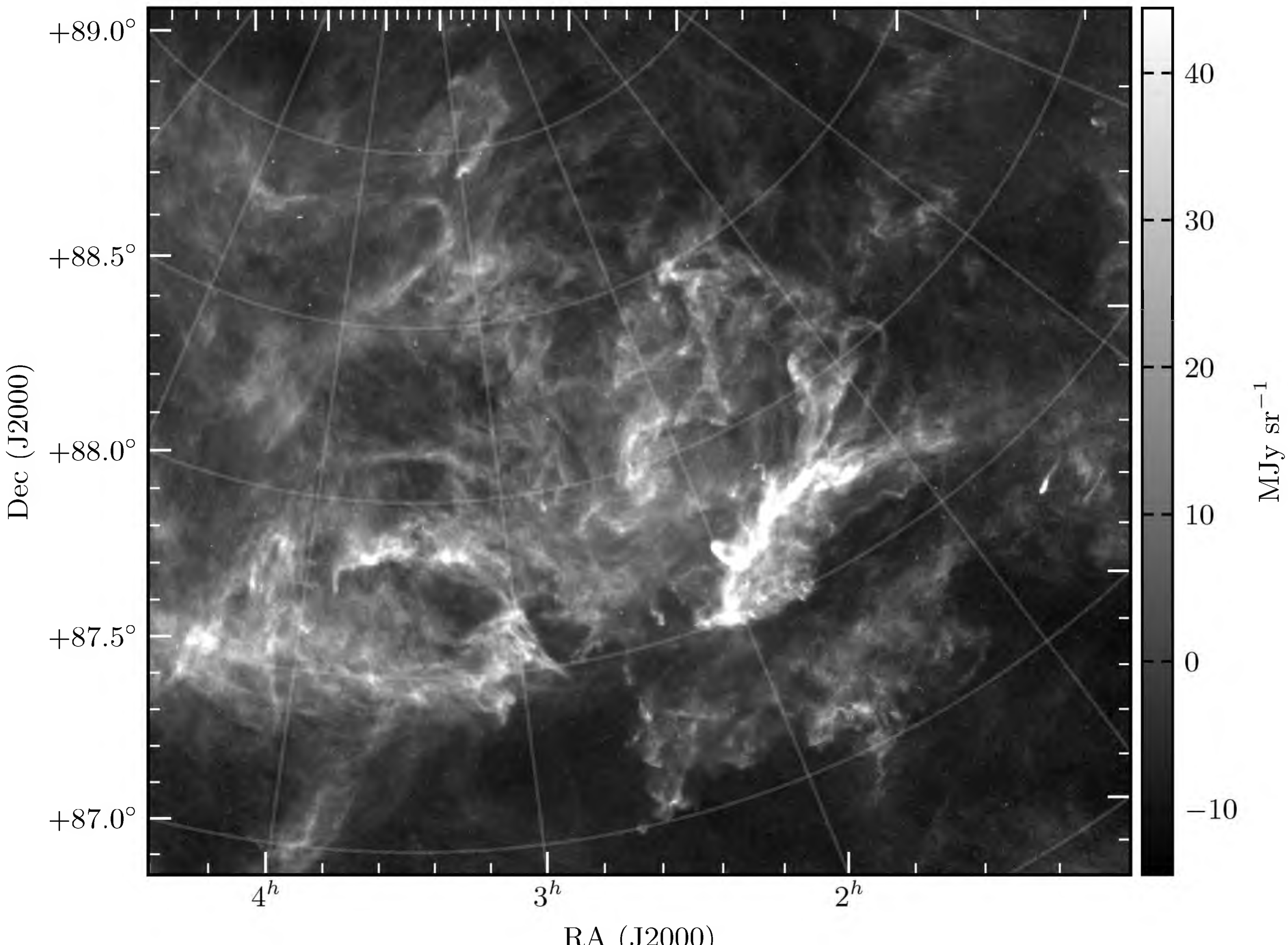
Dec (J2000)
4h
3h
2h
RA (J2000)
40
30
20
10
0
−10
MJy sr−1

Effect described by Elmegreen et al. 2001(HI); Miville-Deschênes et al. 2007 (IR)

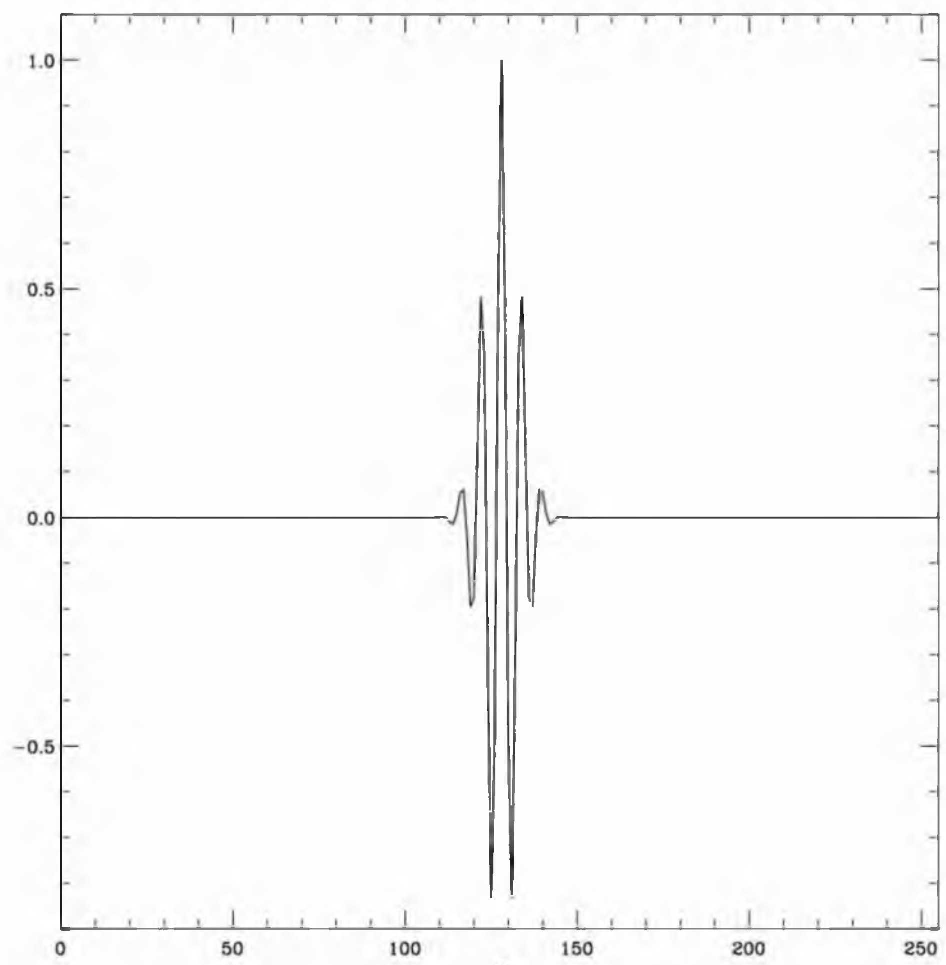


# Complex Wavelet transform

$$\psi(\mathbf{x}) = e^{i\mathbf{k}_0\mathbf{x}}e^{-|\mathbf{x}|^2/2}$$
$$= [\cos(\mathbf{k}_0\mathbf{x}) - i\sin(\mathbf{k}_0\mathbf{x})]e^{-|\mathbf{x}|^2/2}$$

Morlet wavelet

X space (real part)

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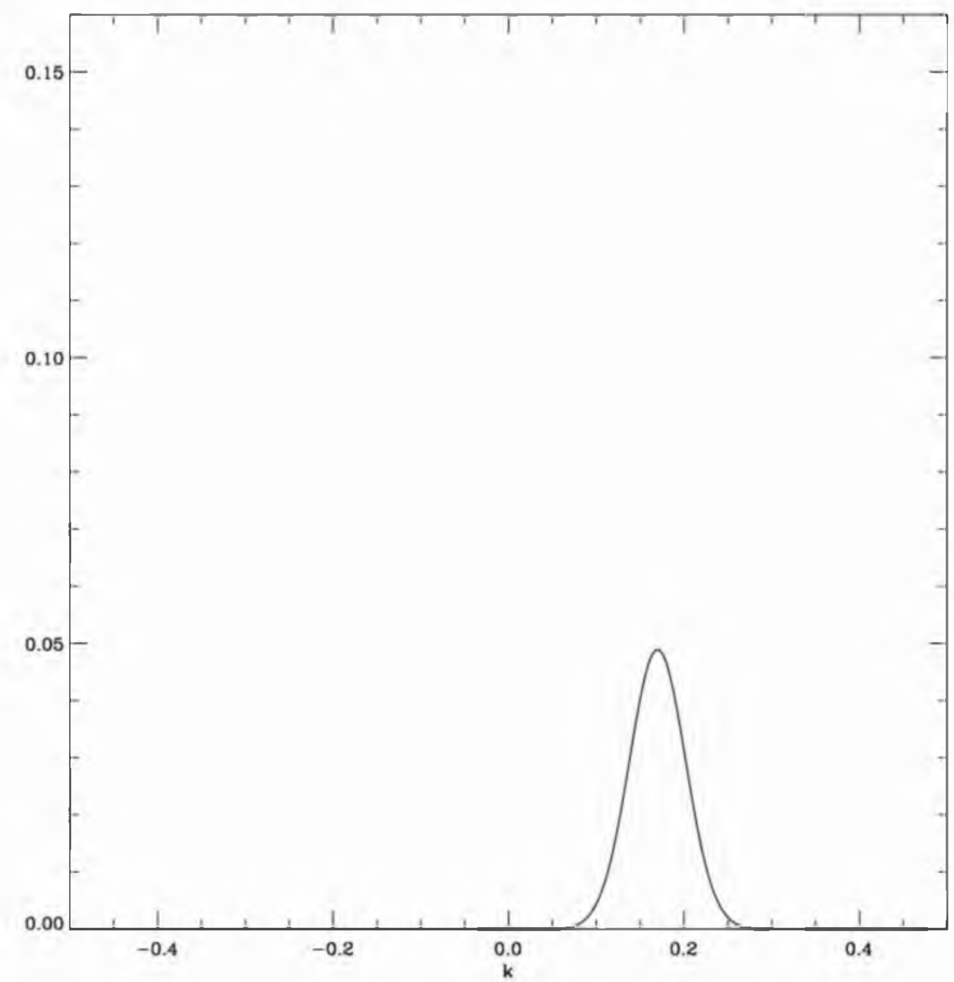

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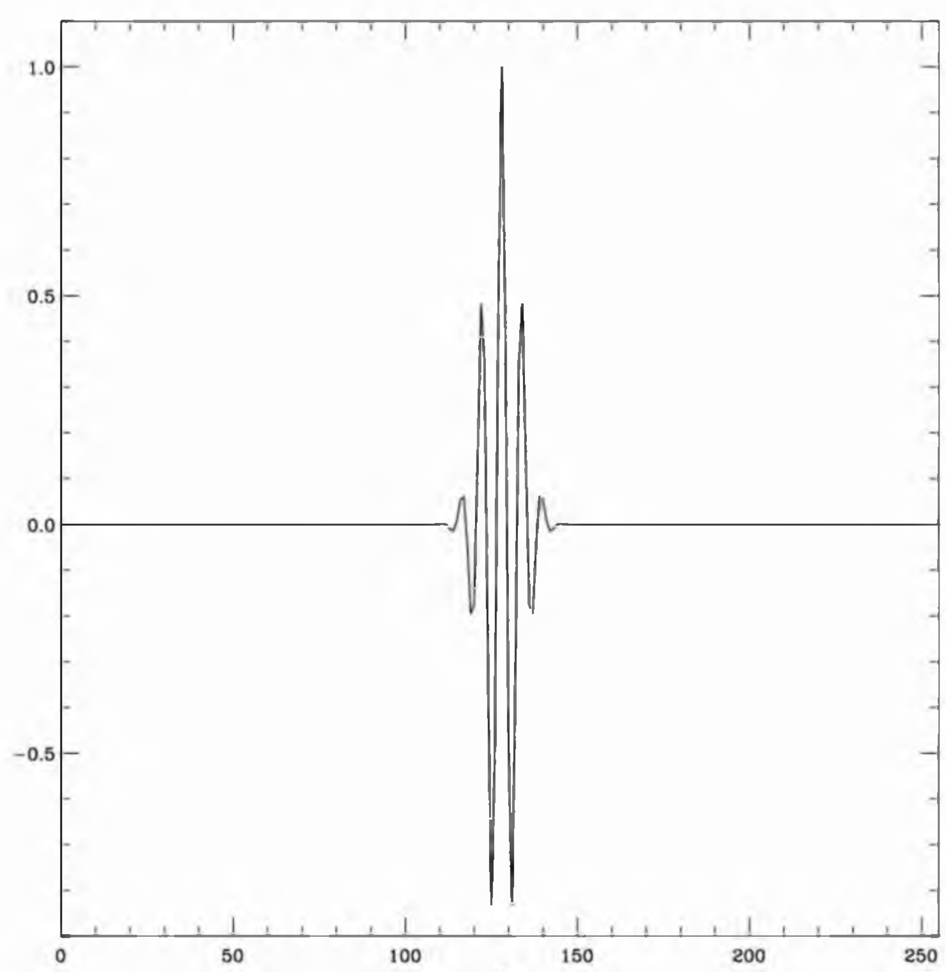


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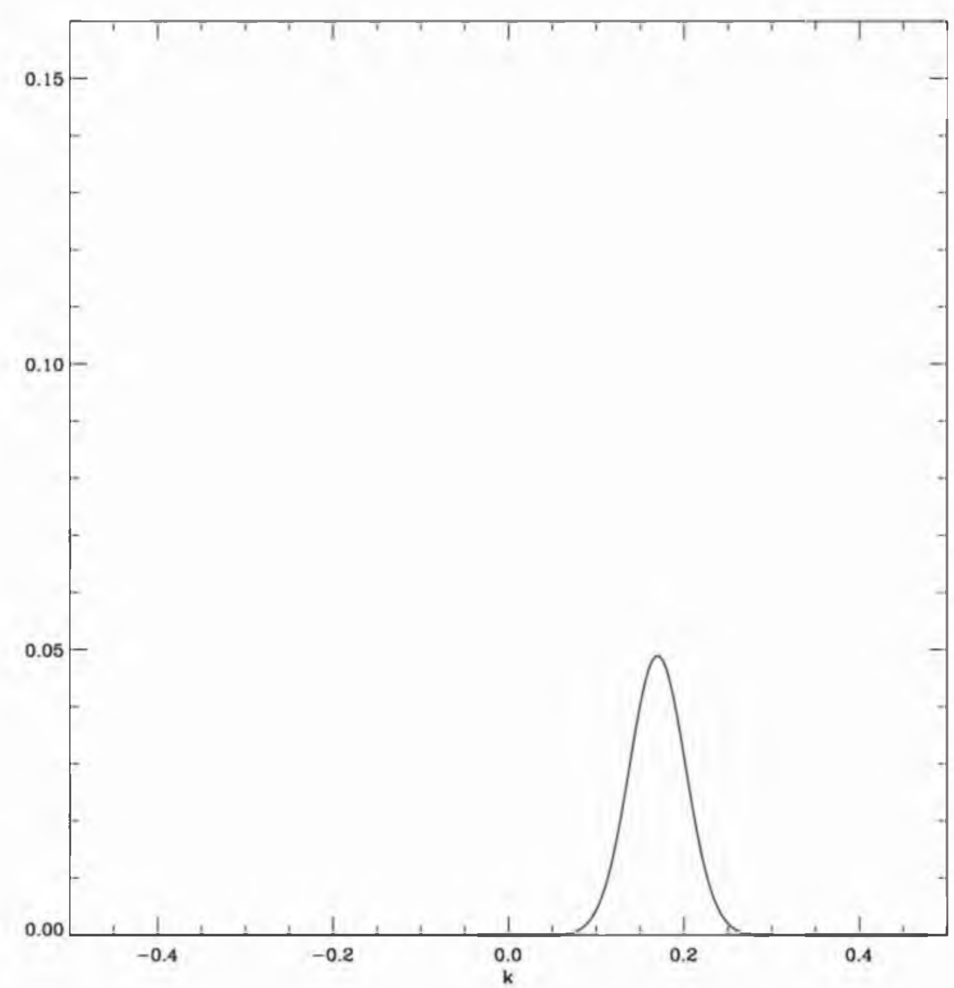

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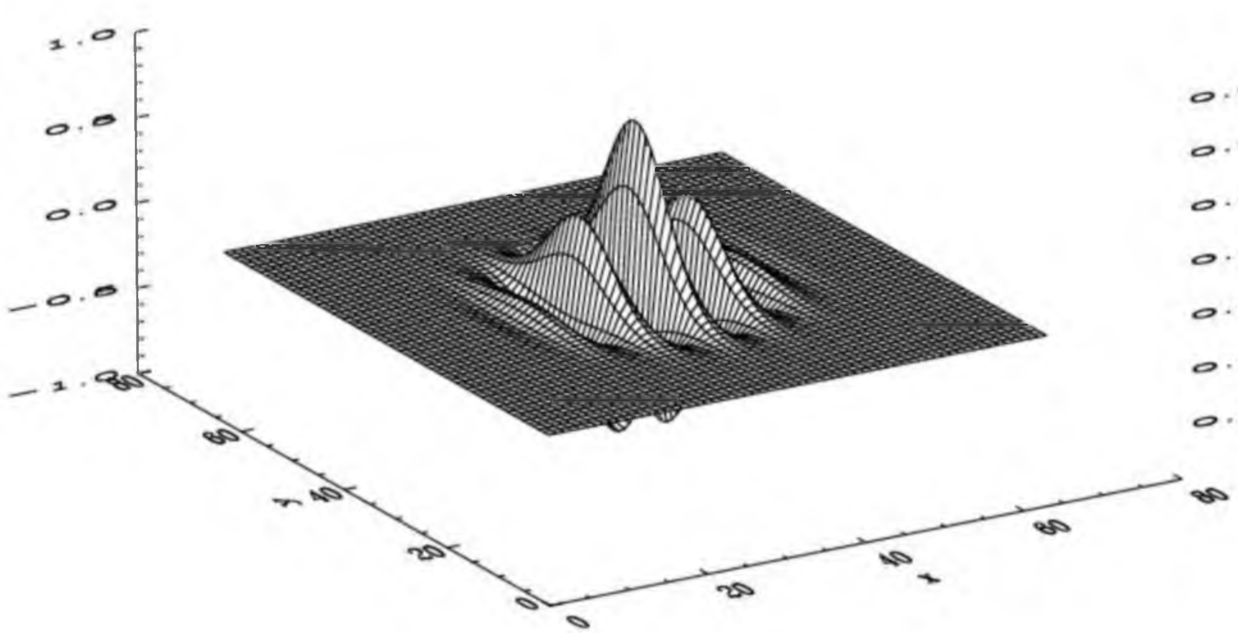

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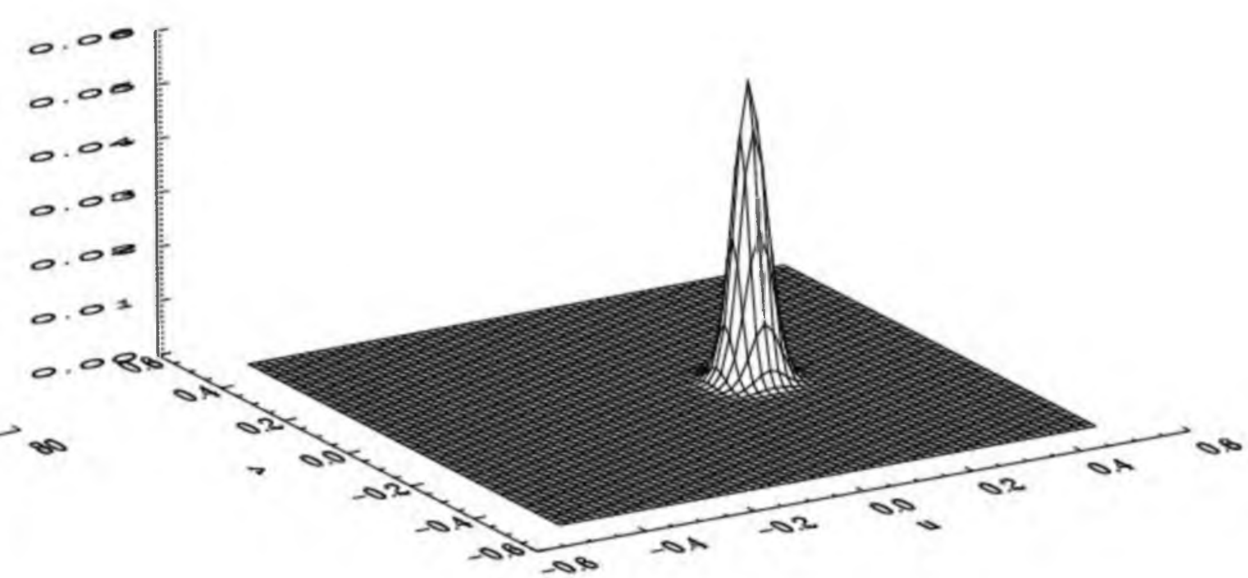

Kirby, J. F. 2005, Computers and Geosciences, 31, 846

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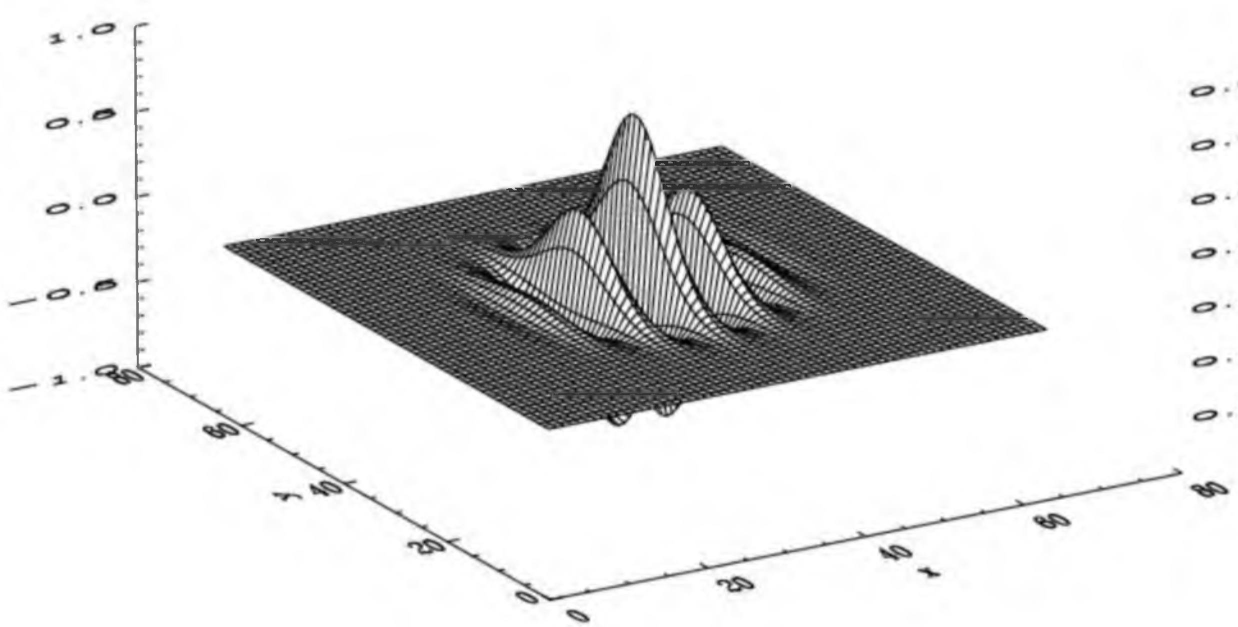

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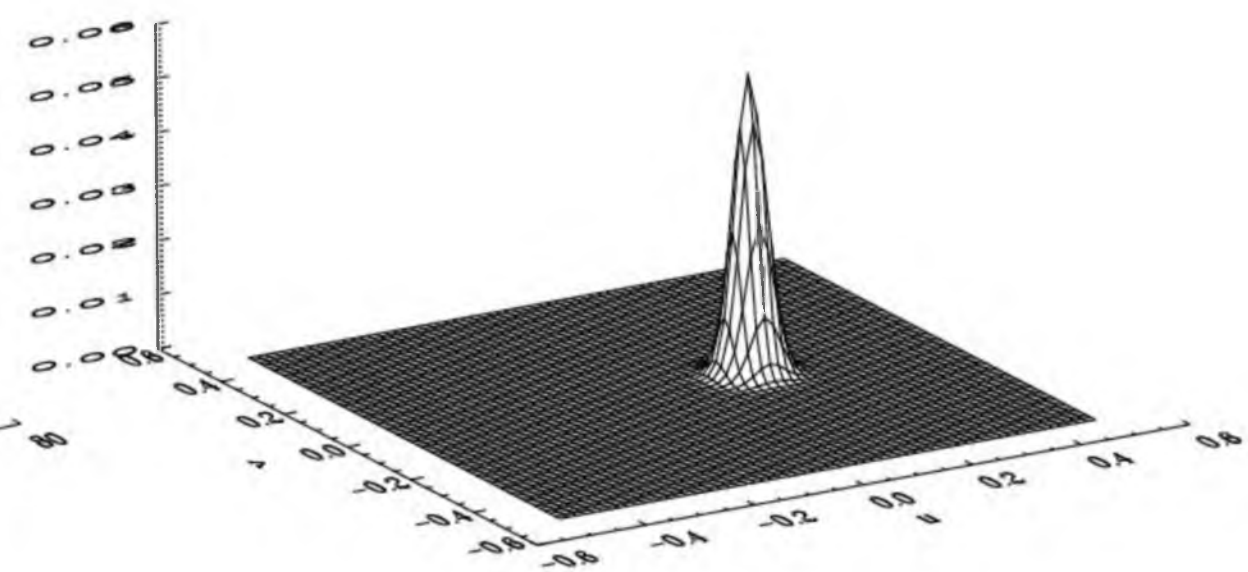

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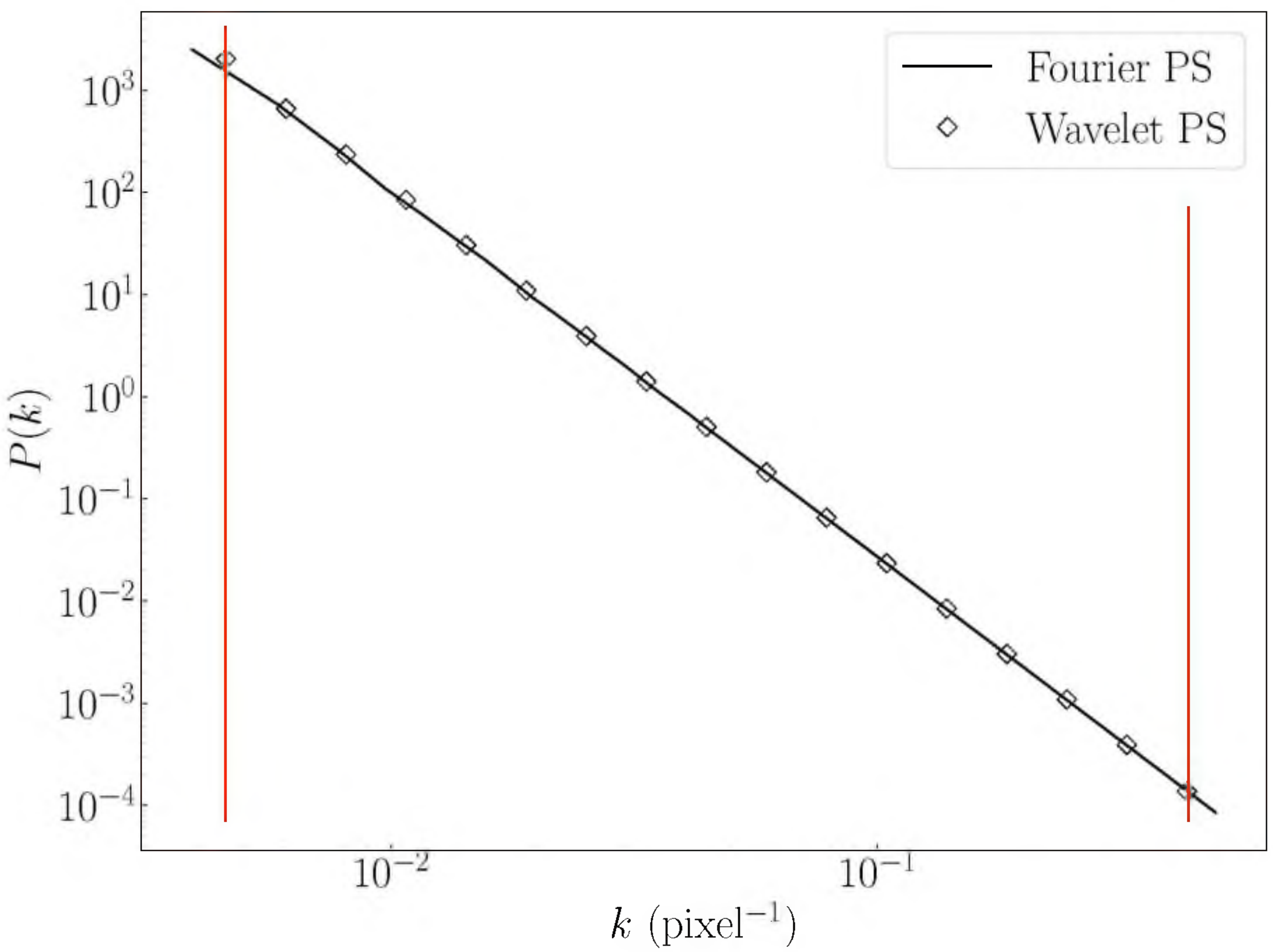


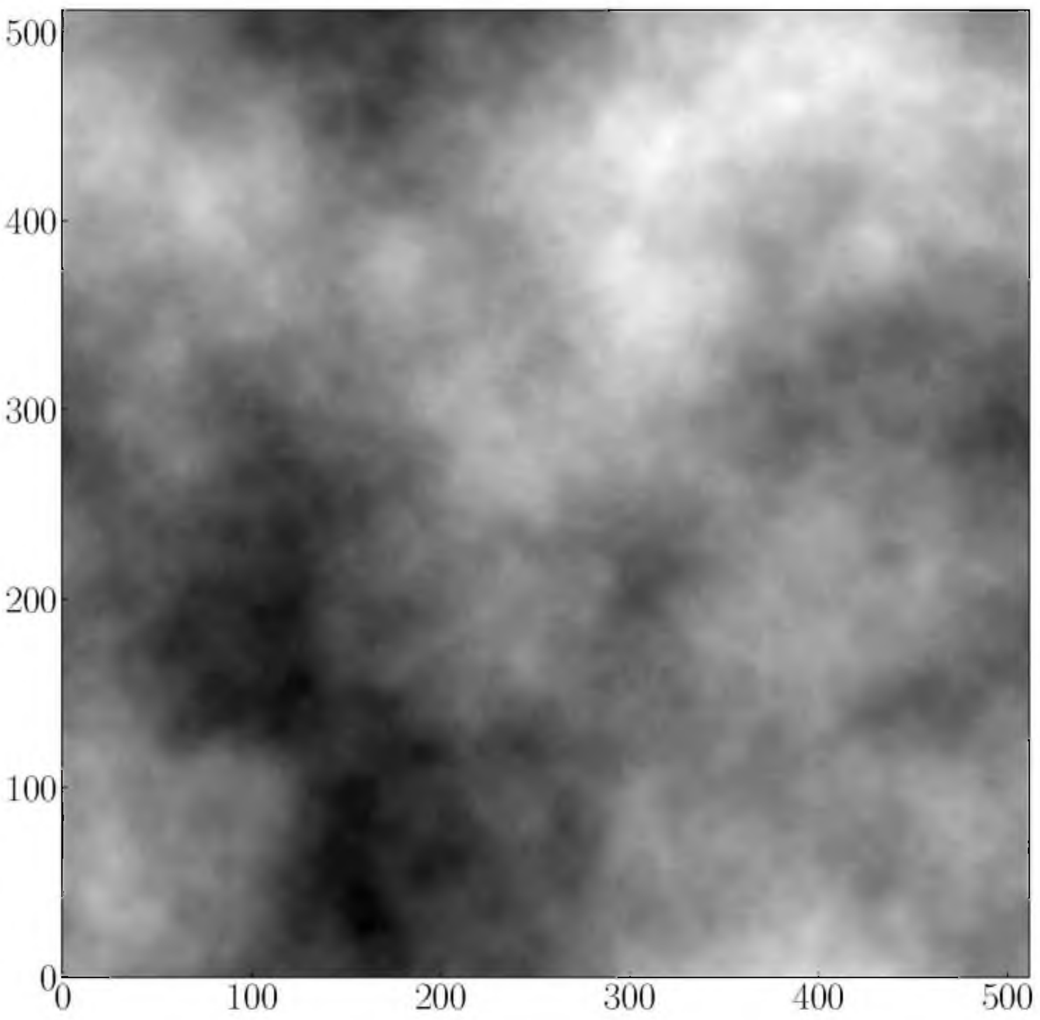

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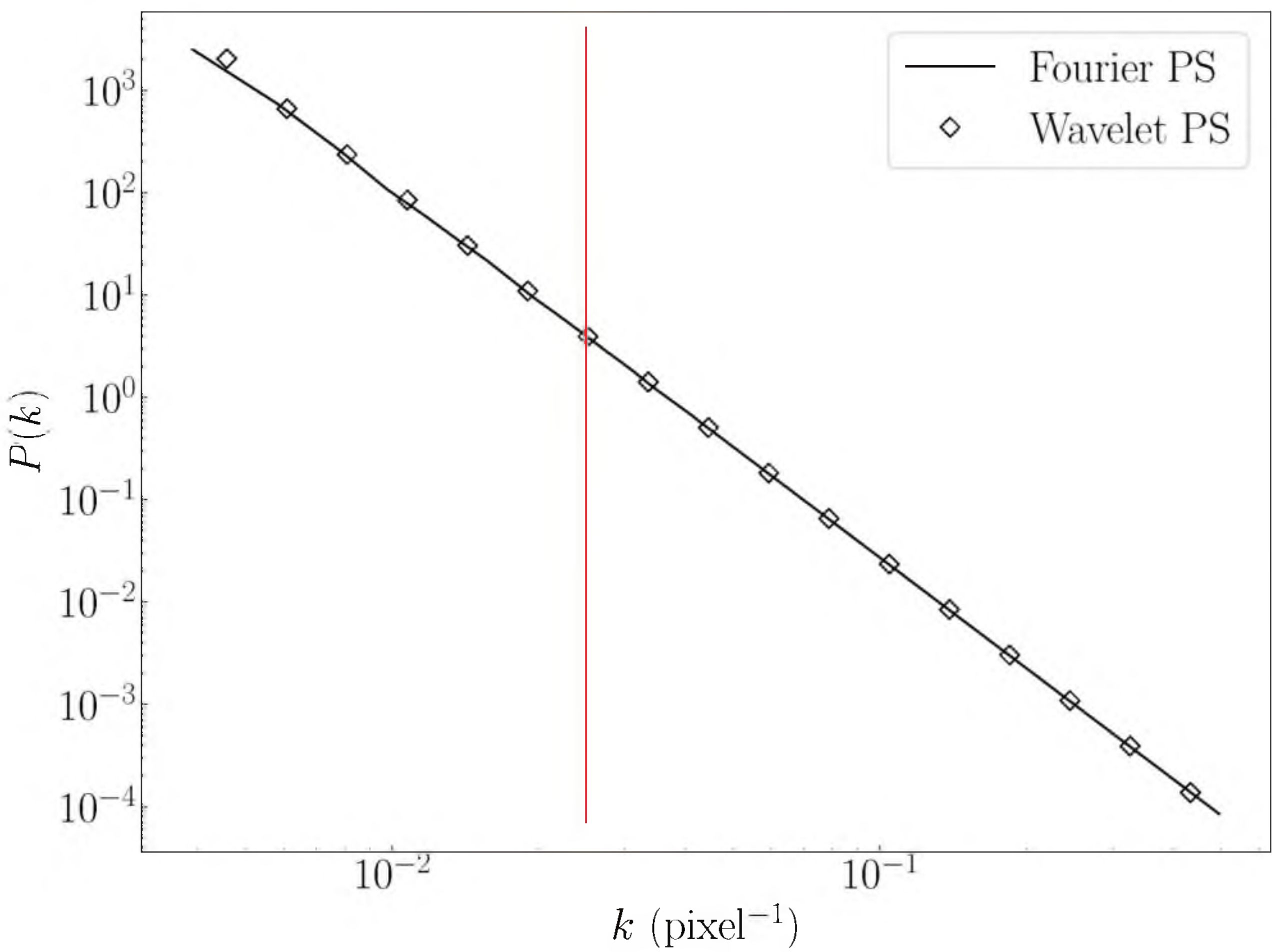


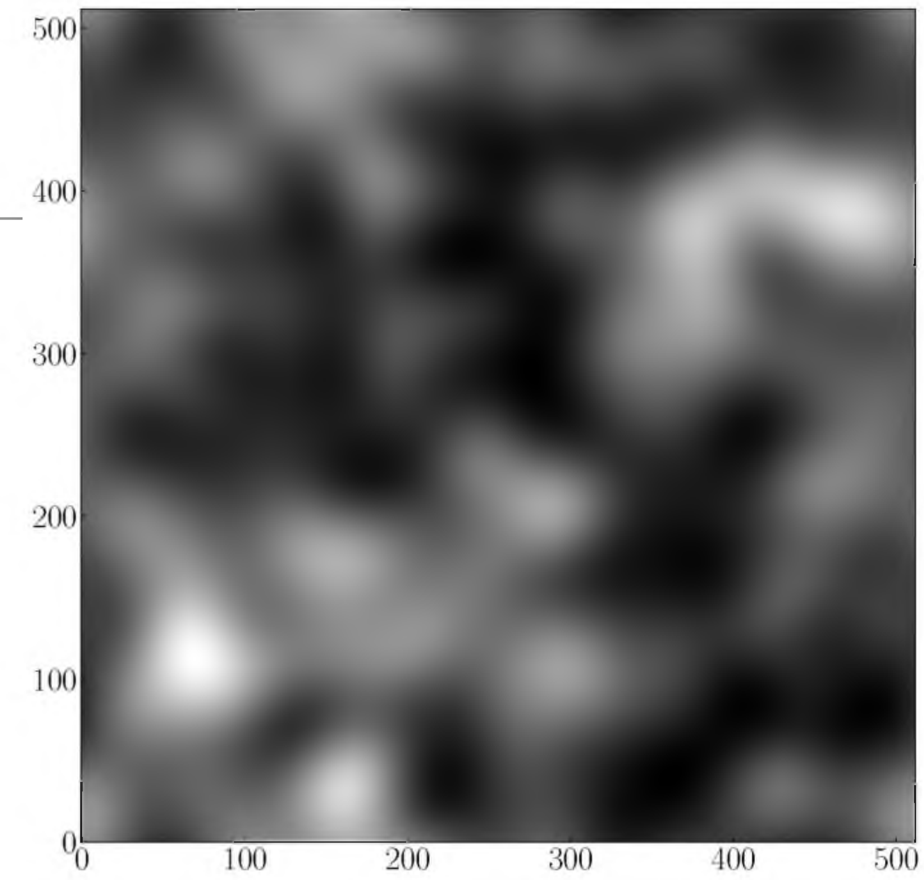


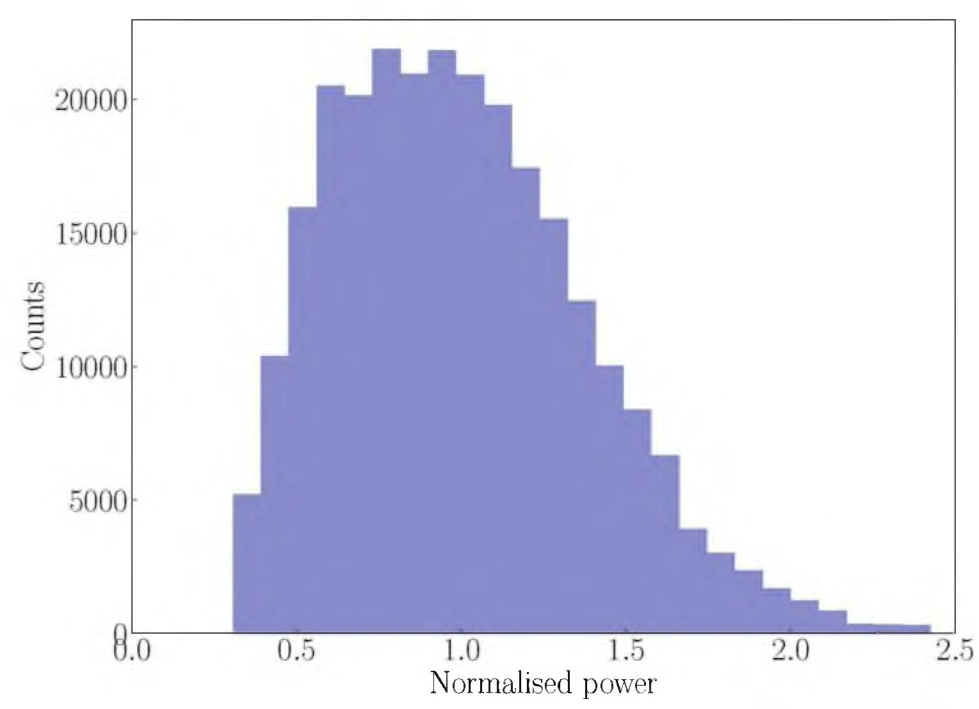

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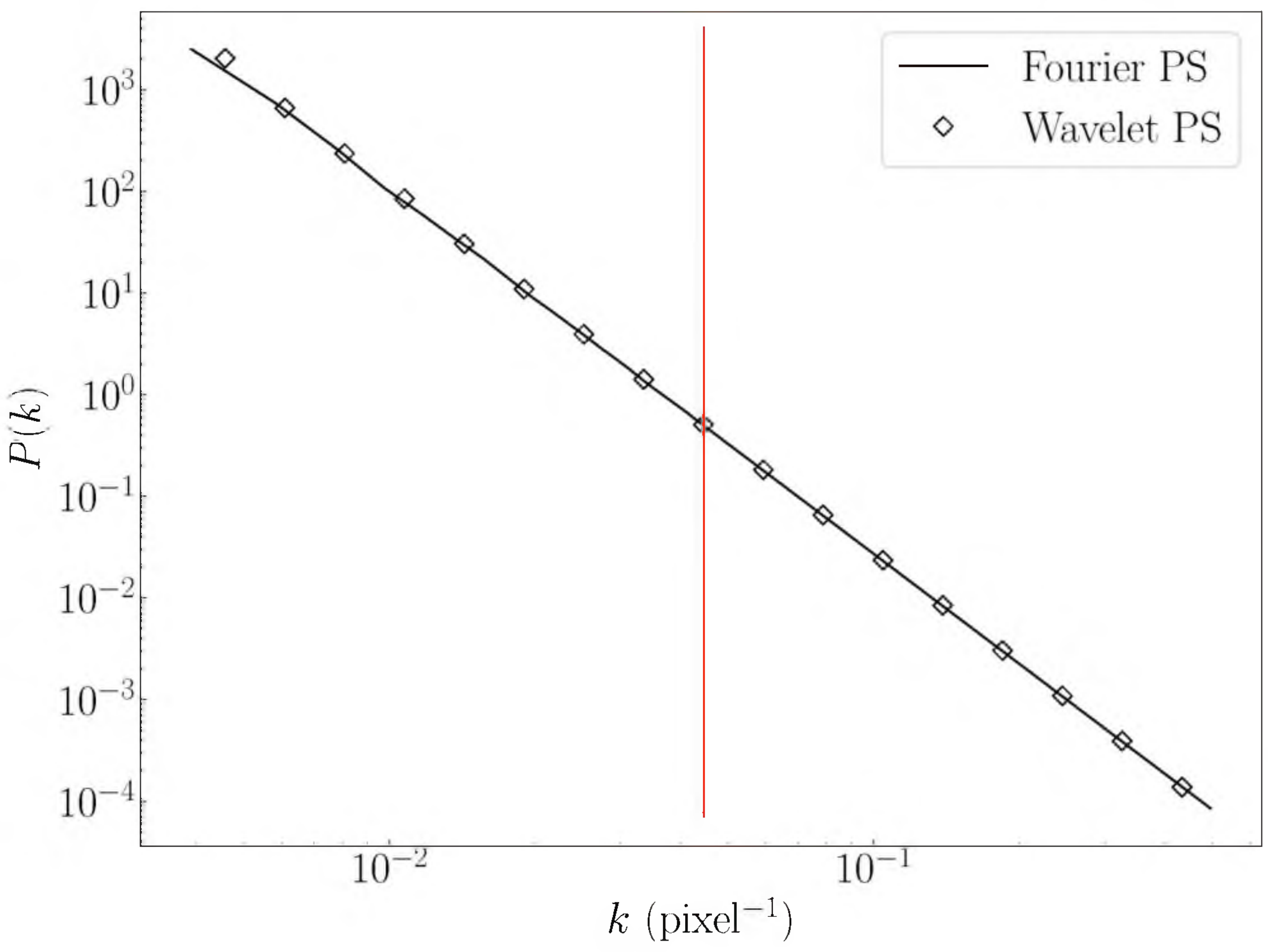


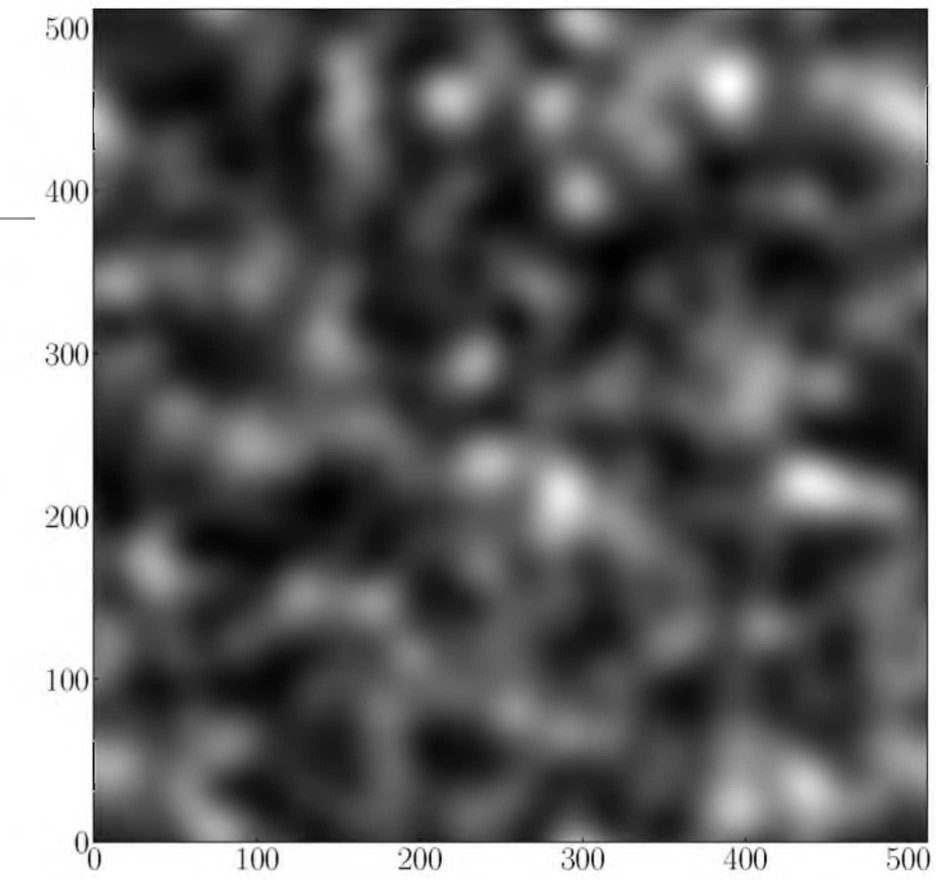


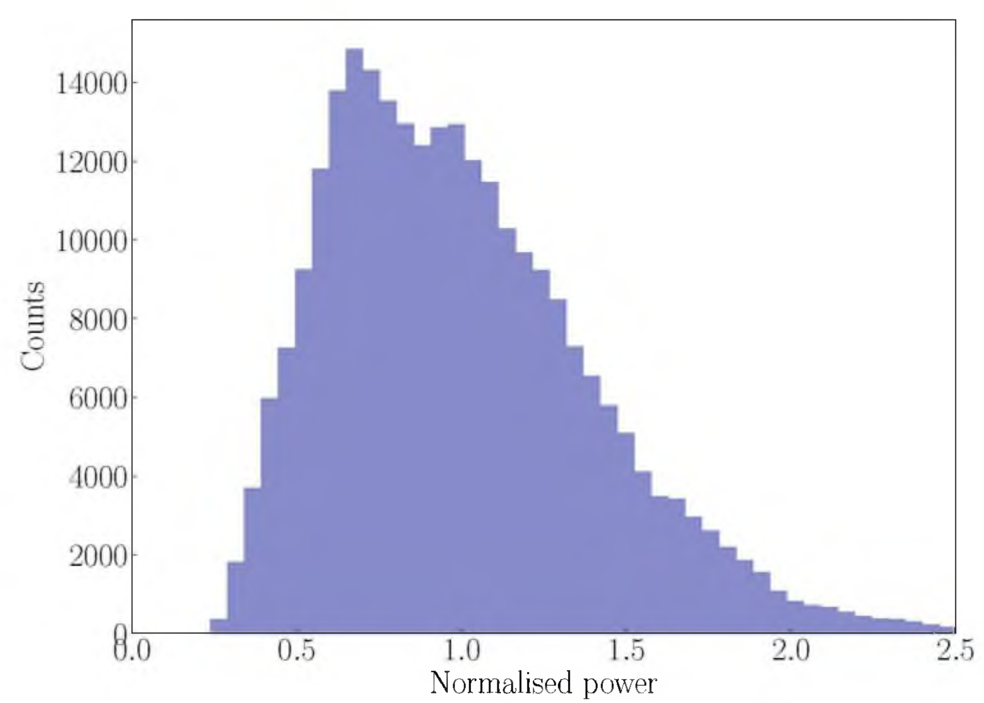

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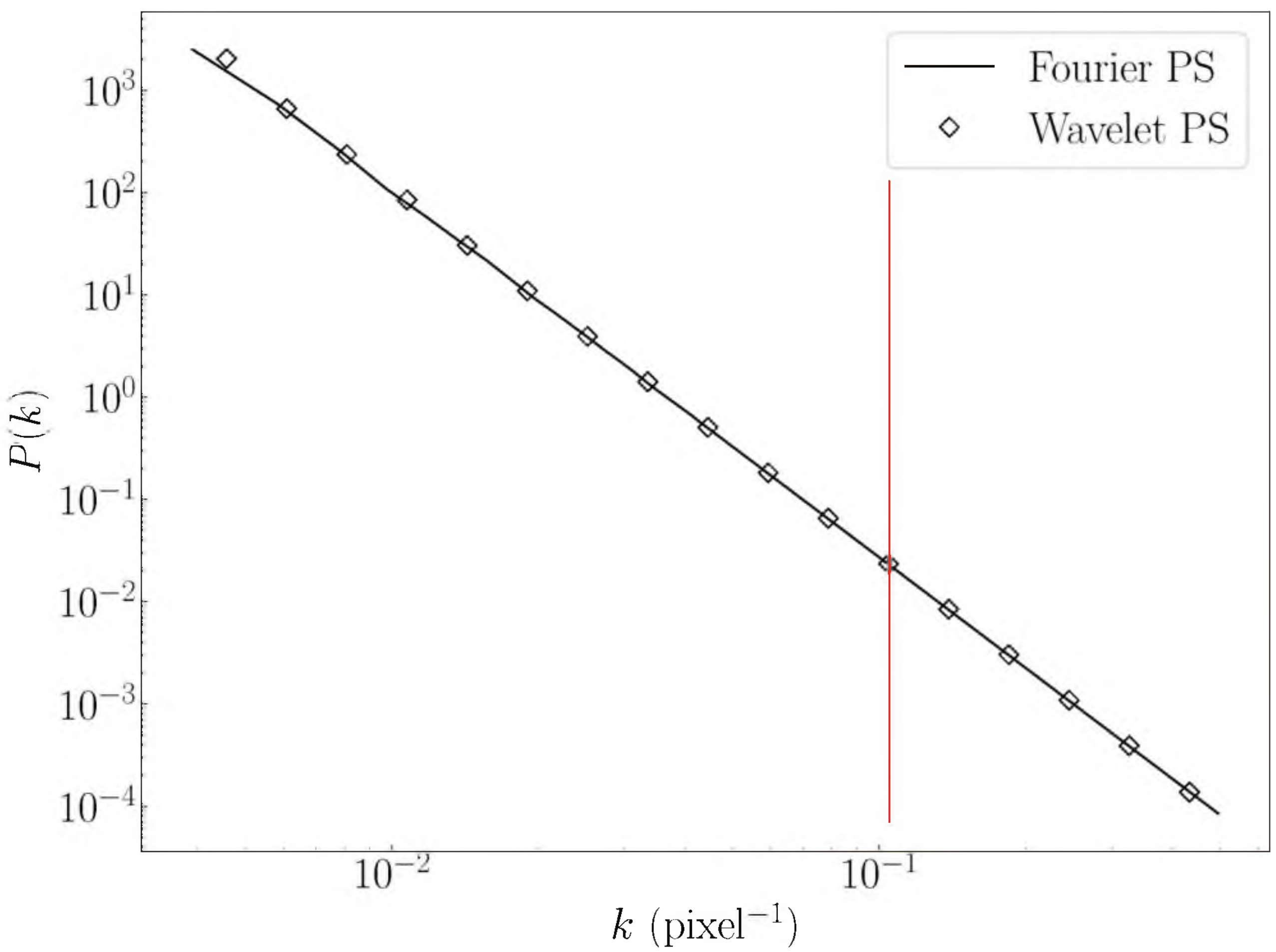


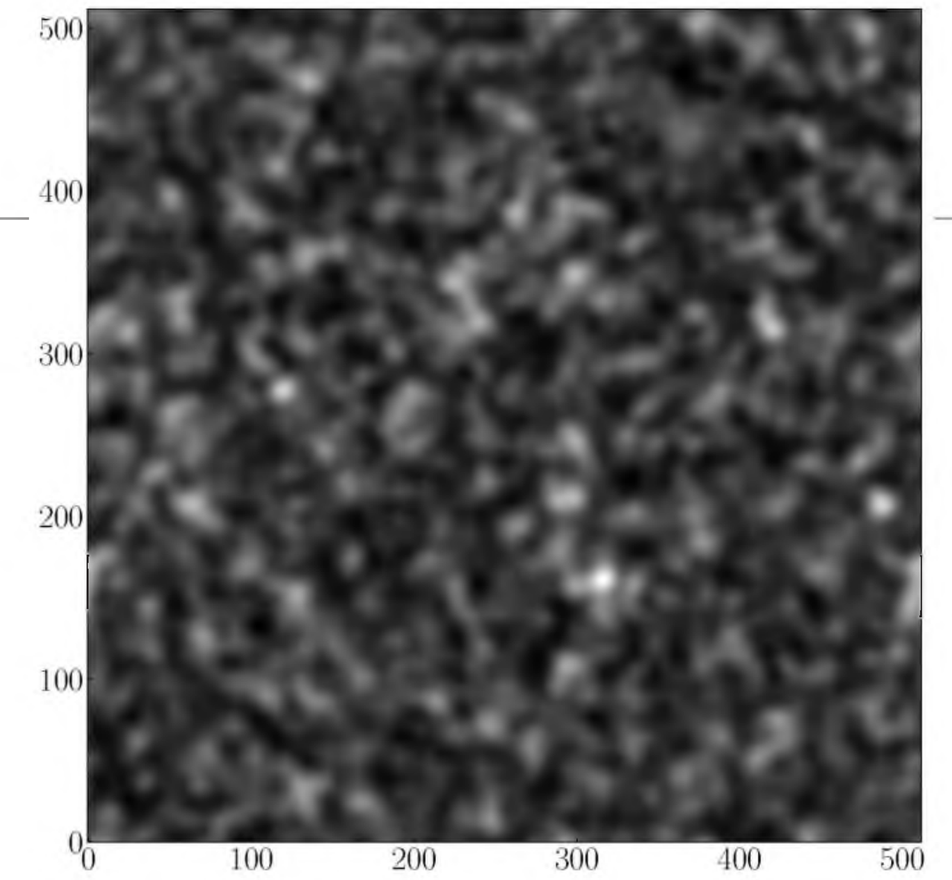

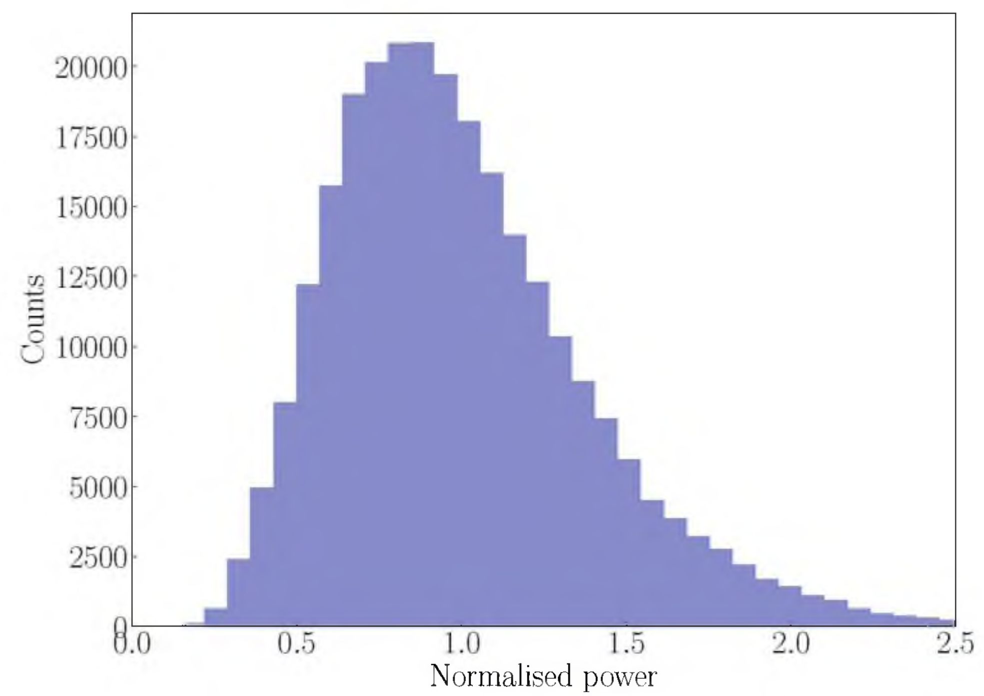

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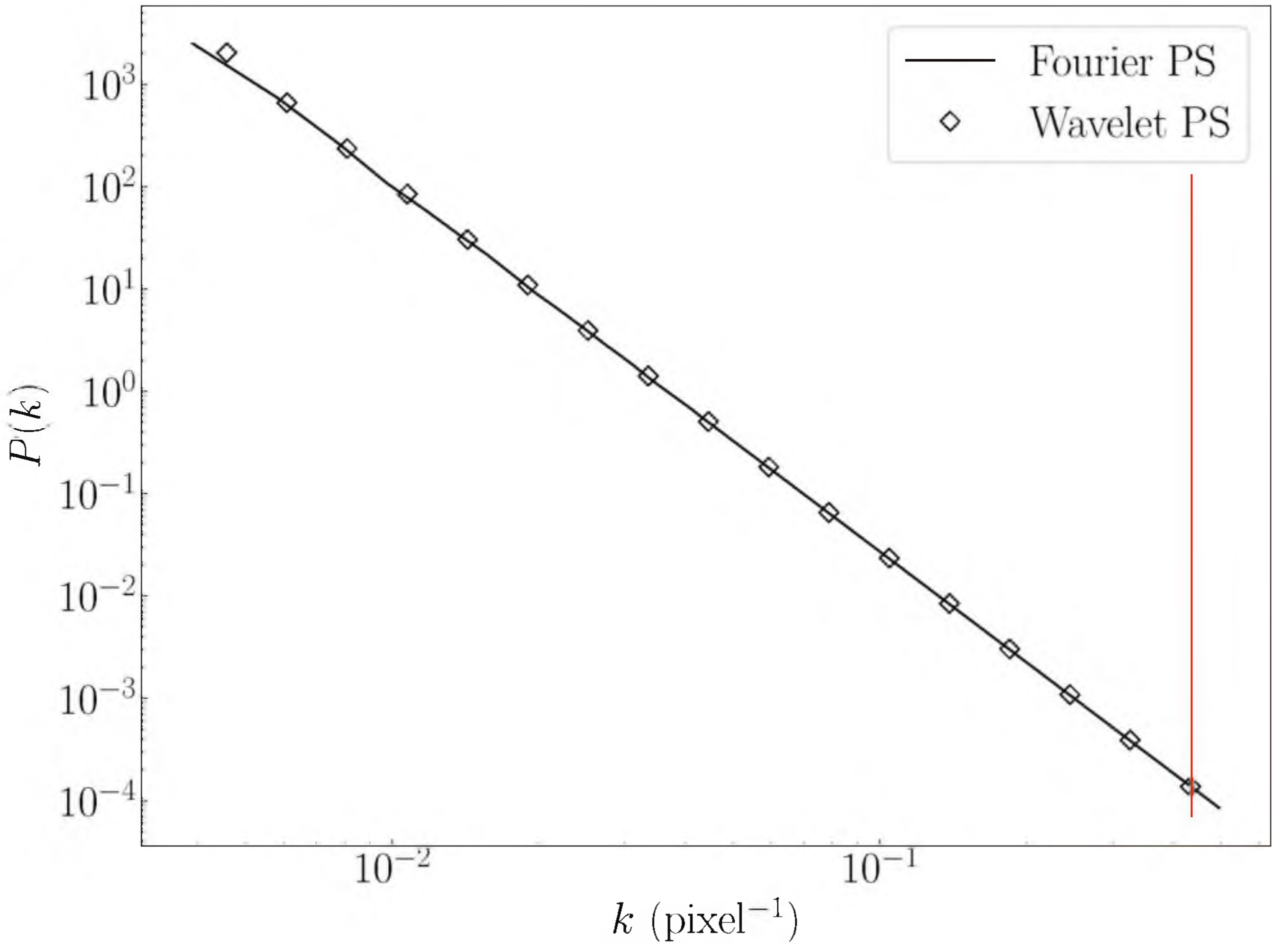
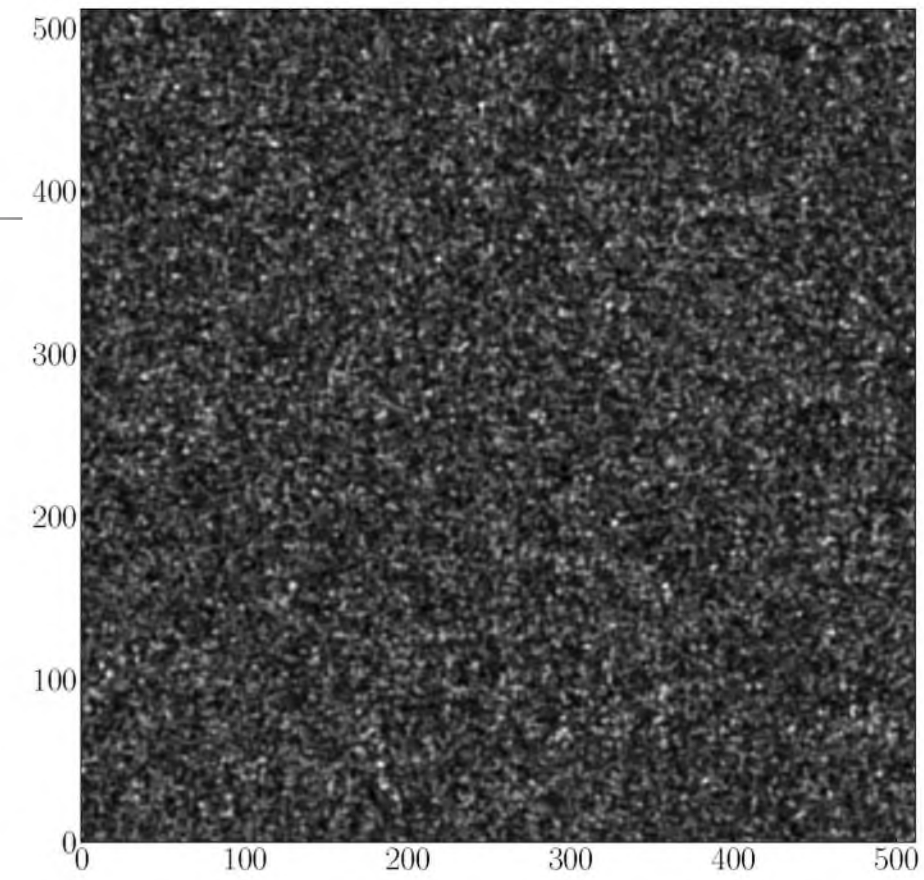
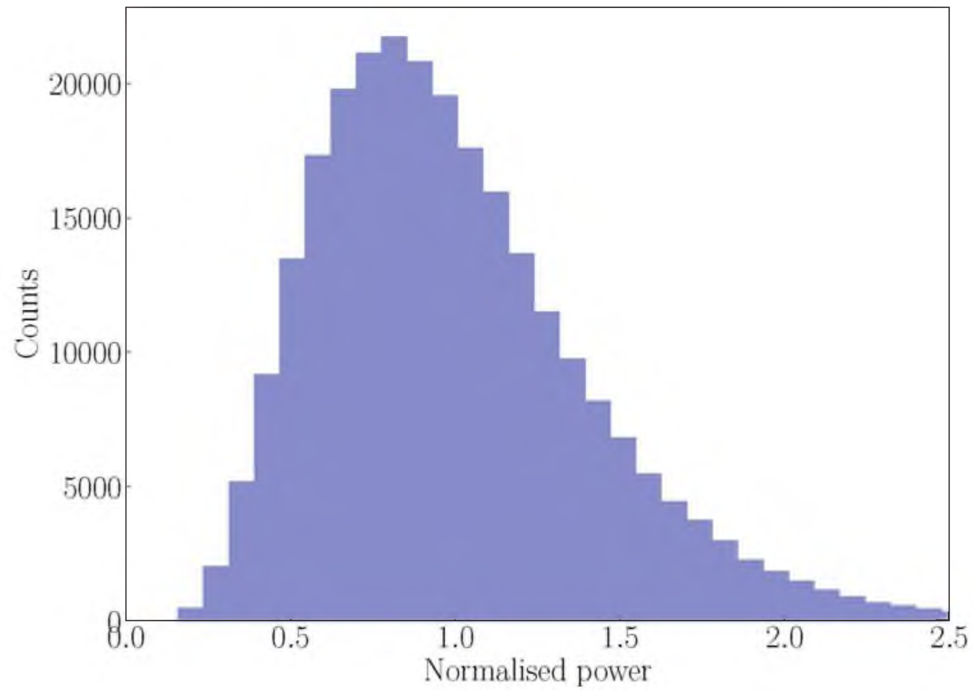
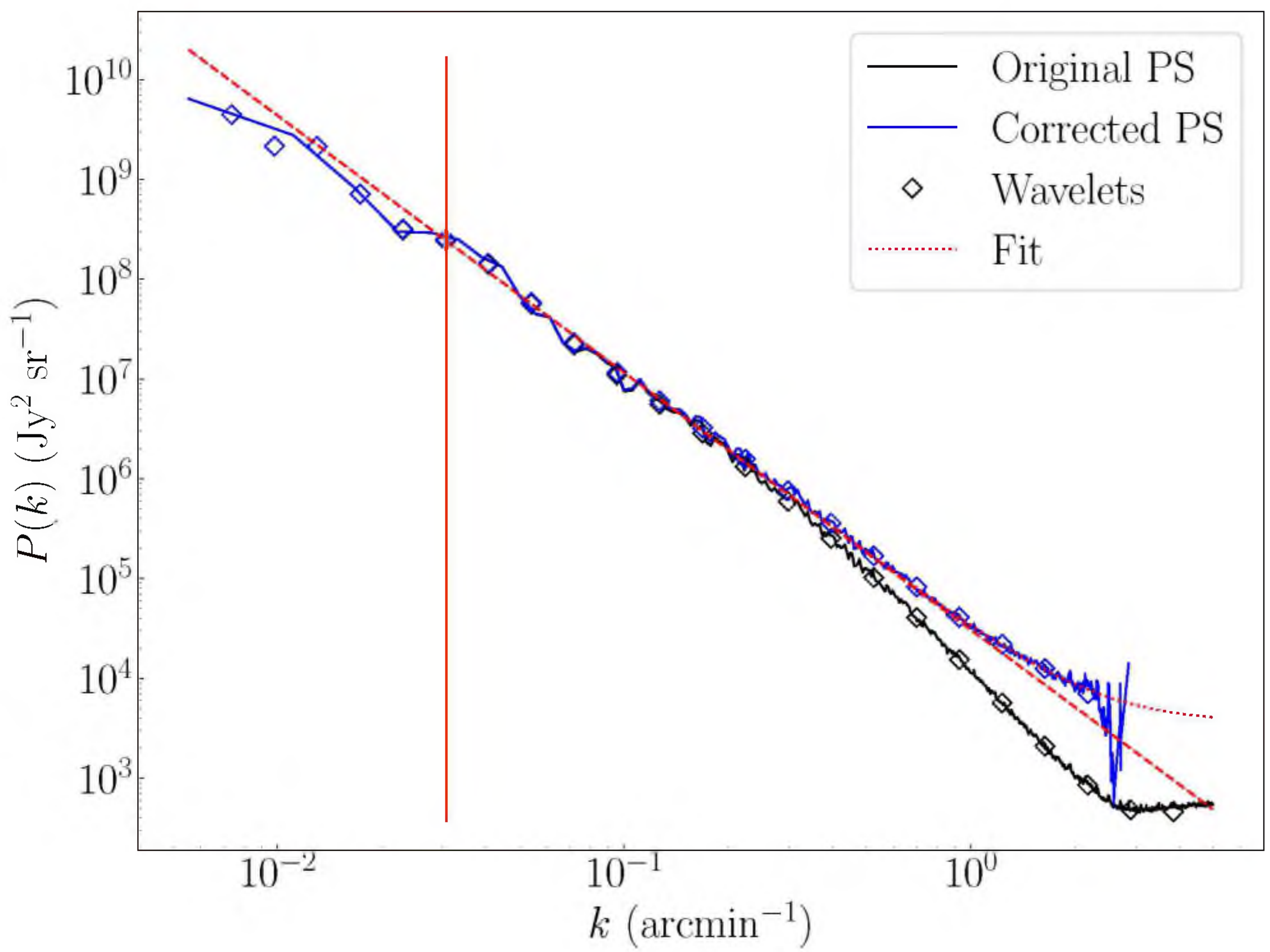
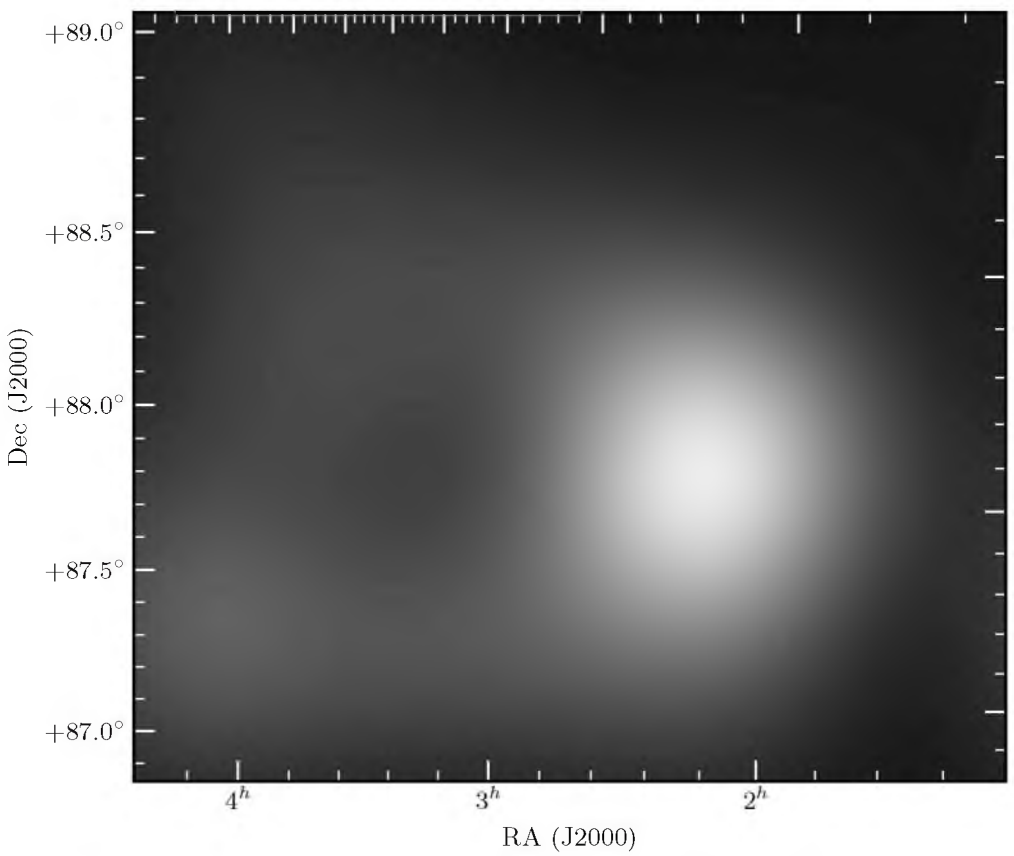


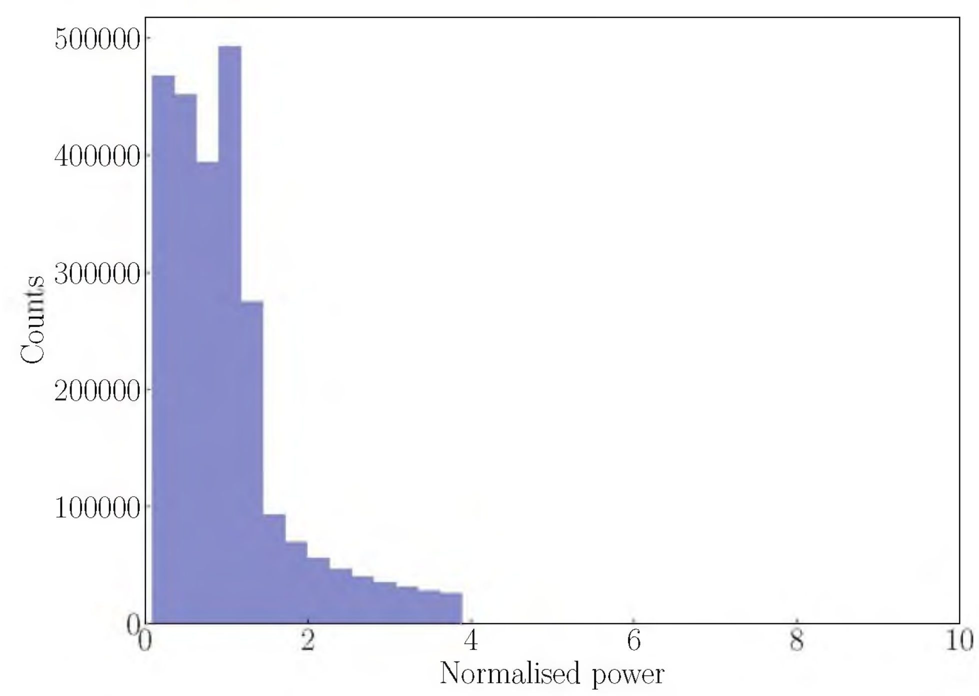



# Polaris 250 µm



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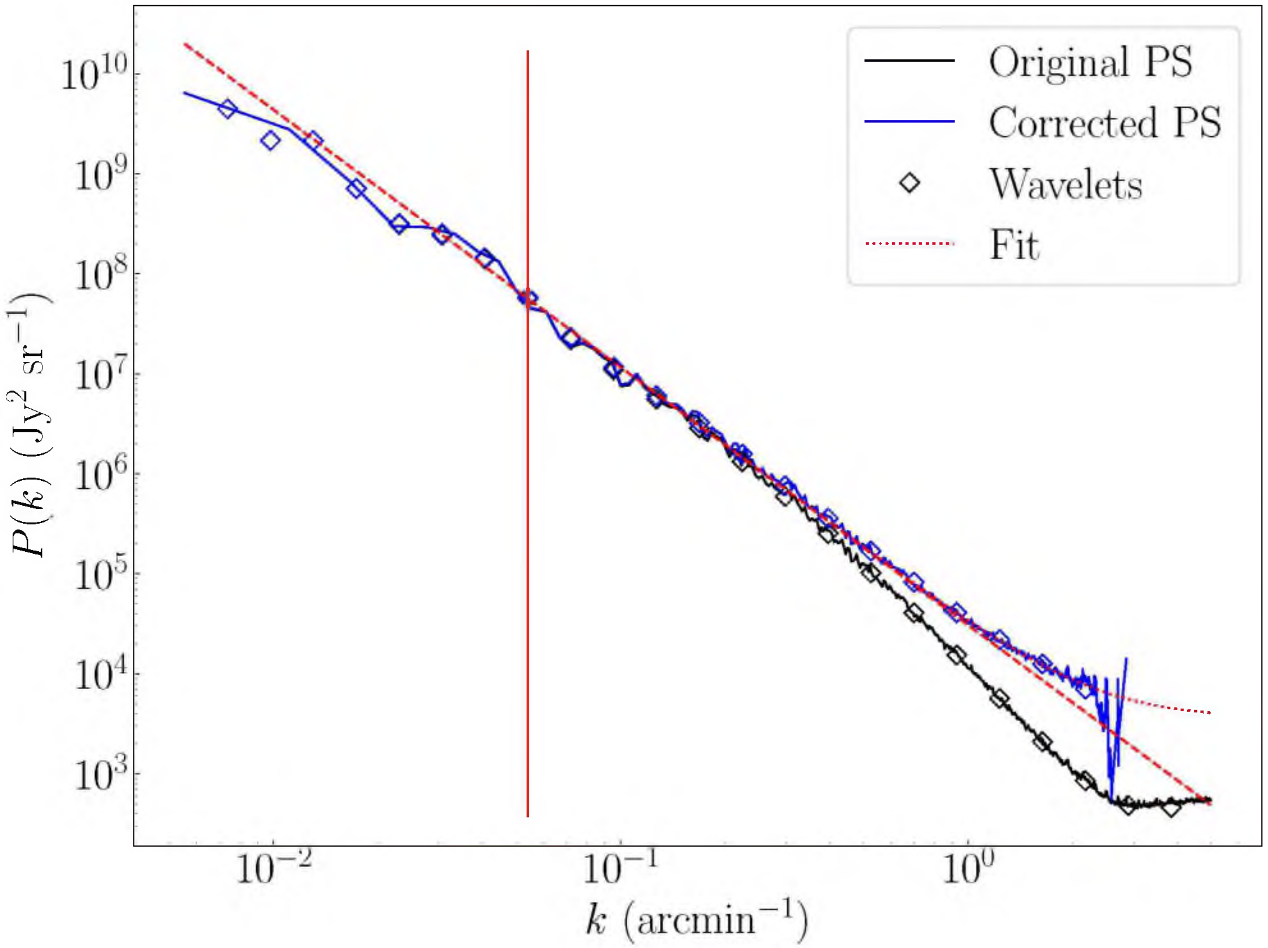


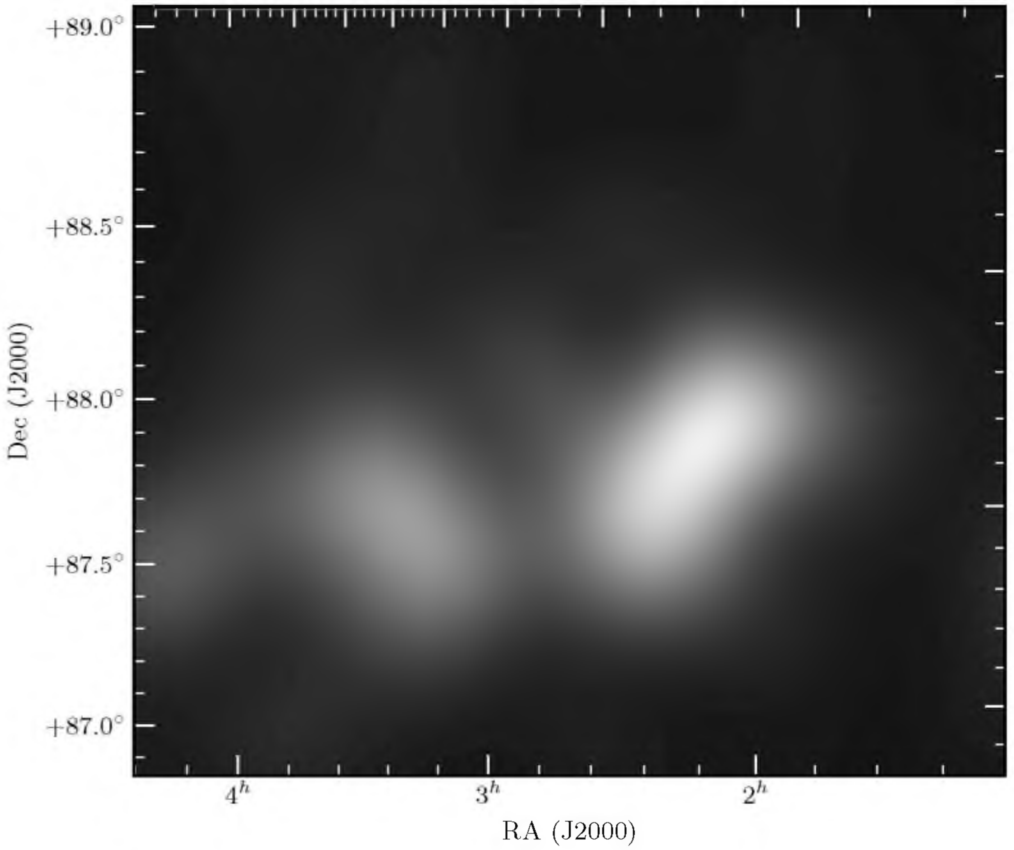


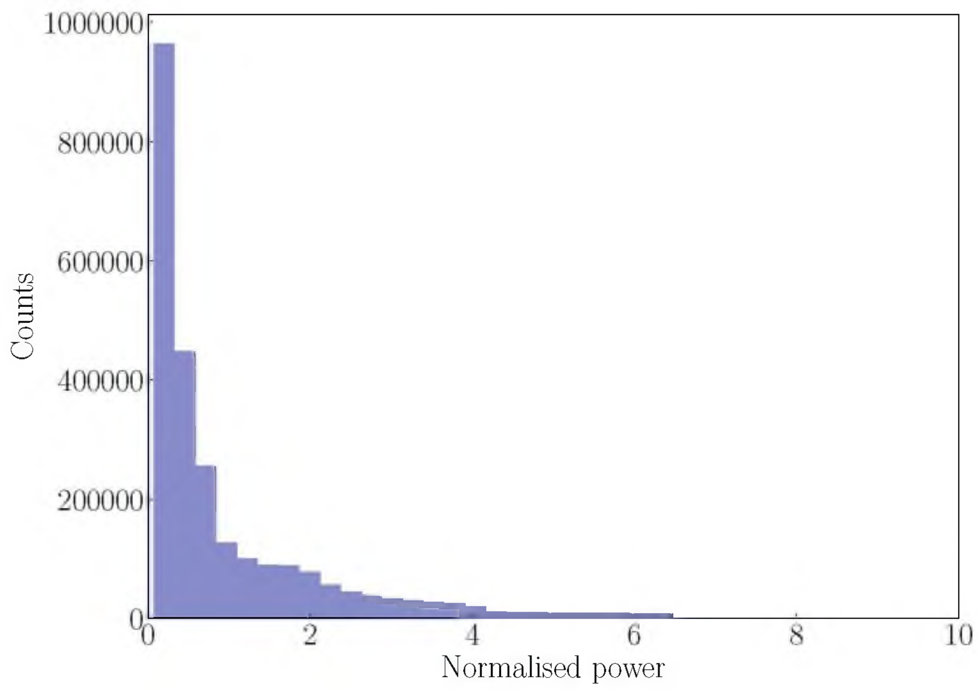

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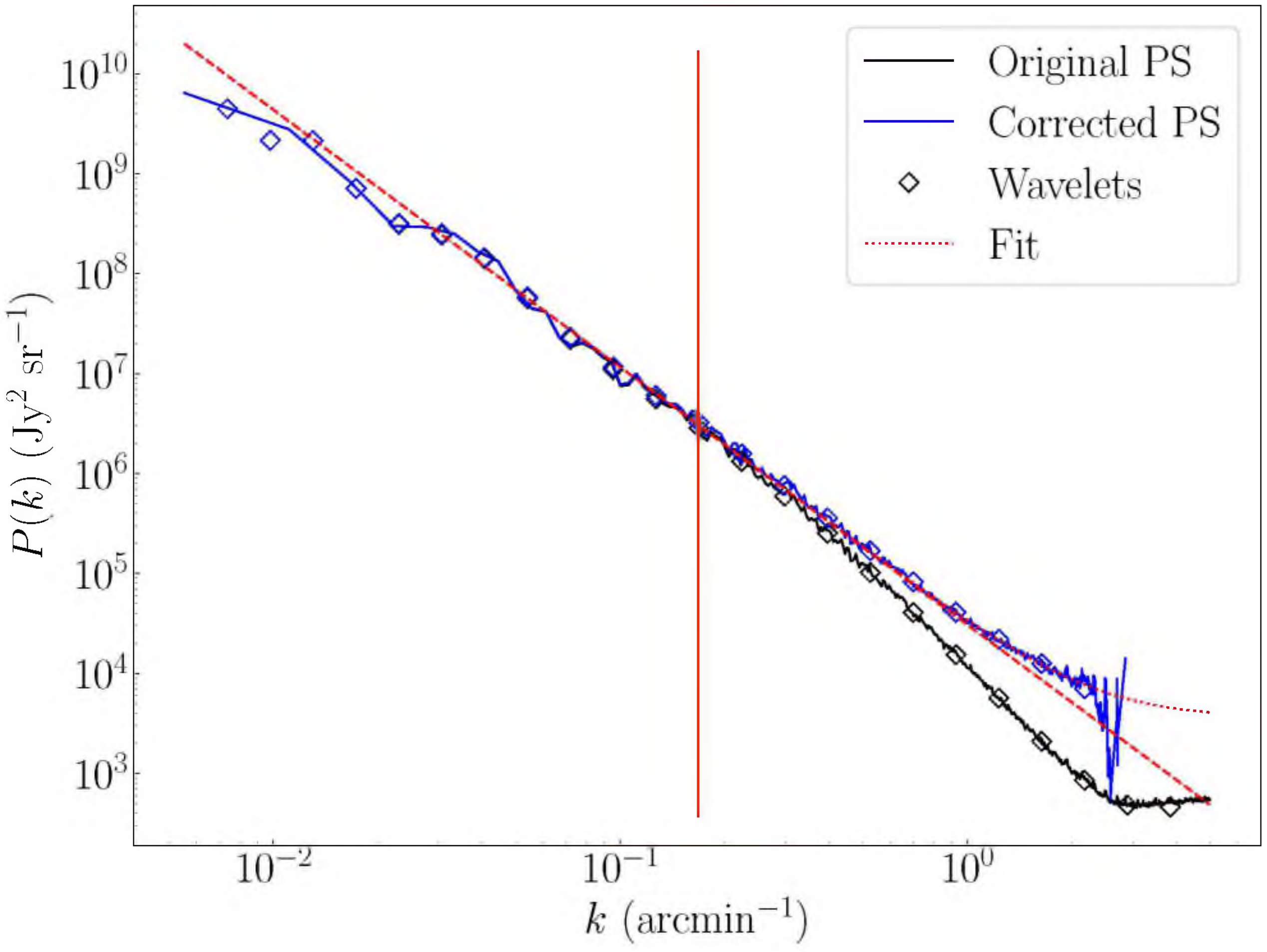


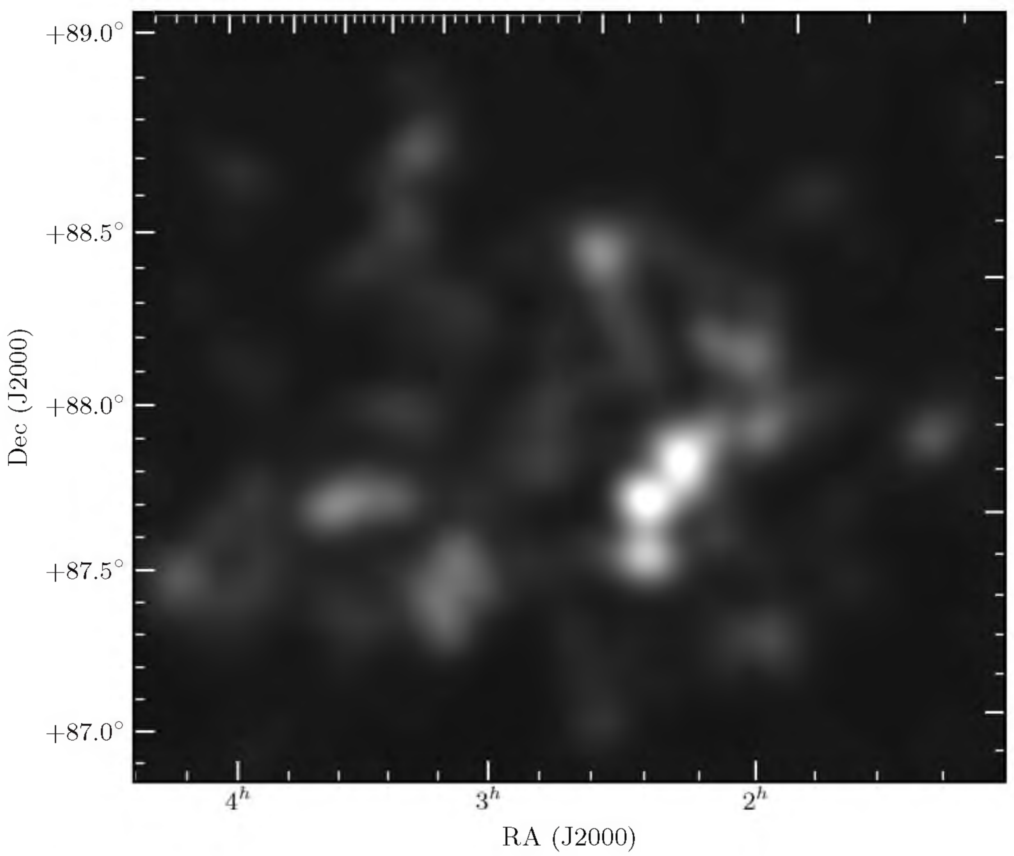


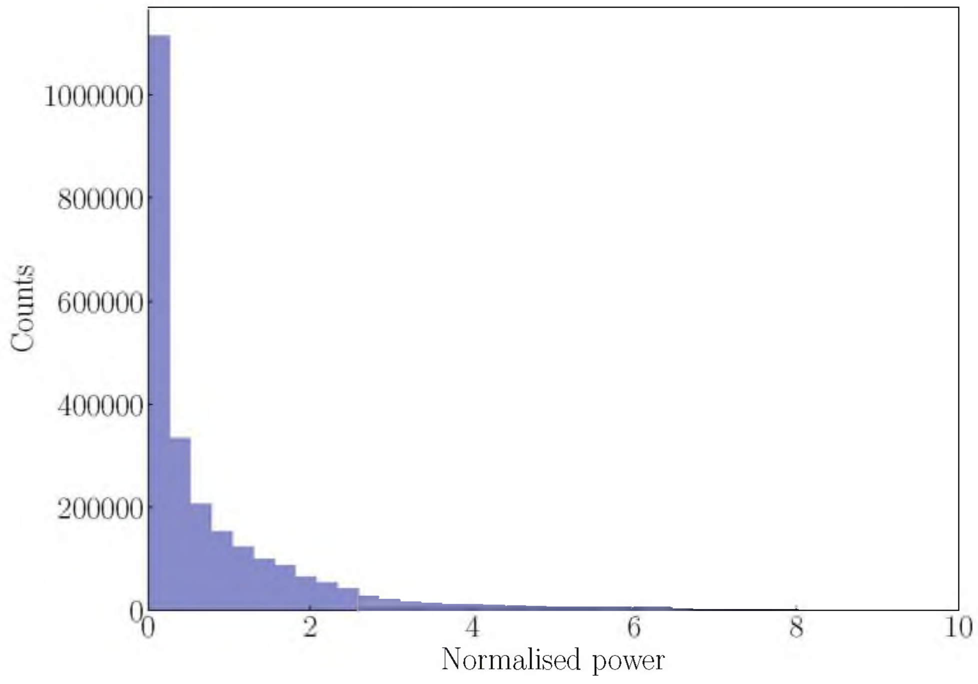

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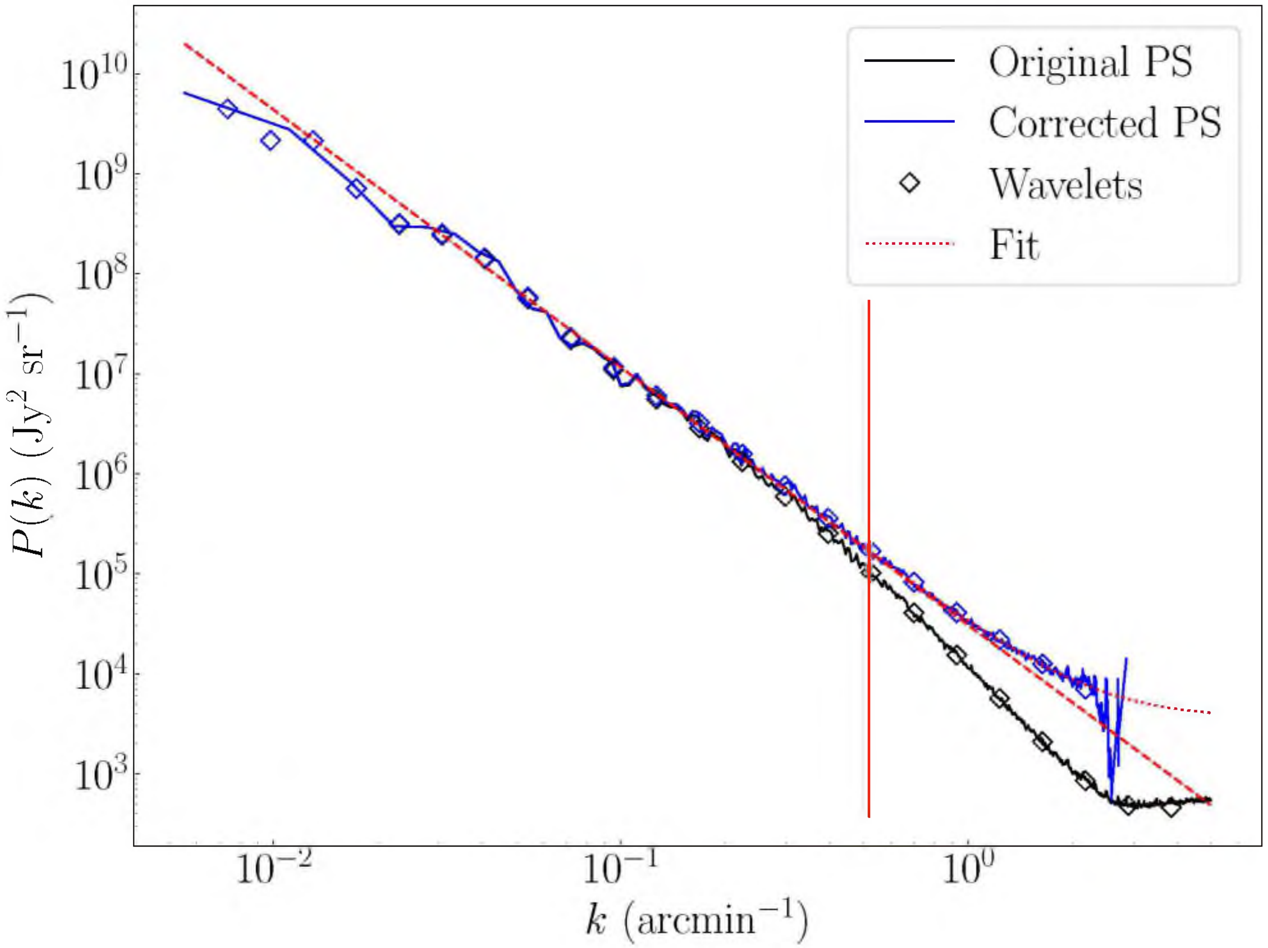
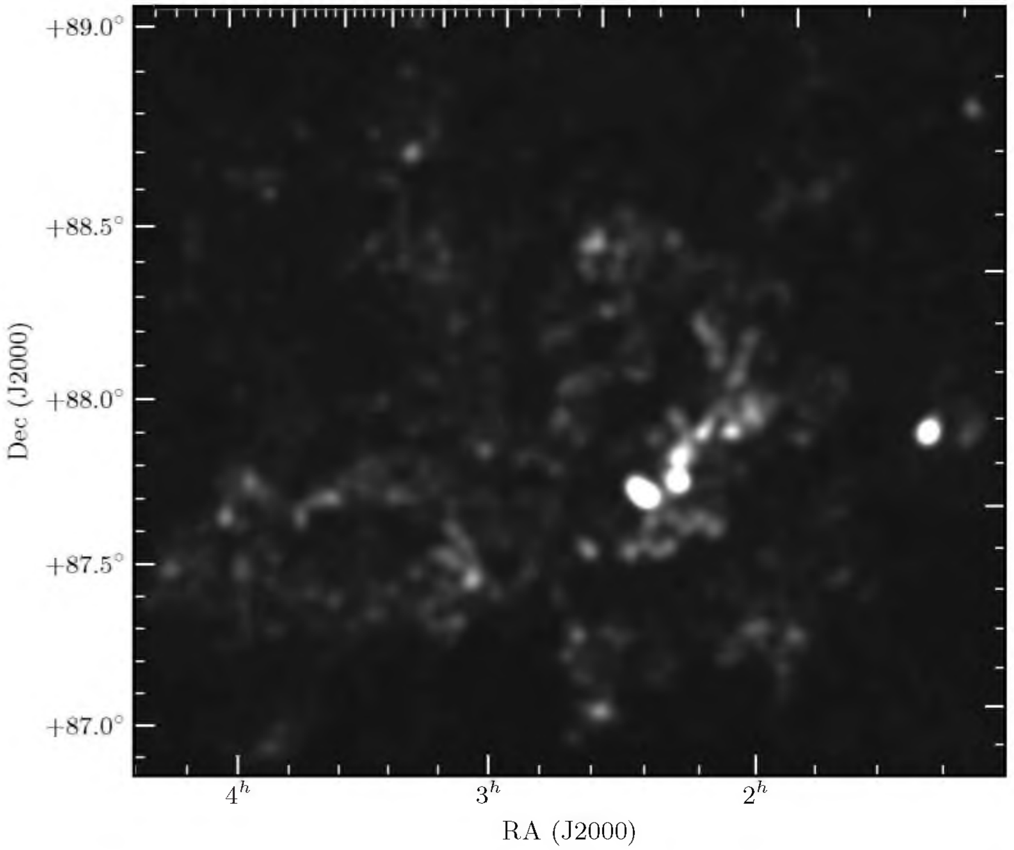
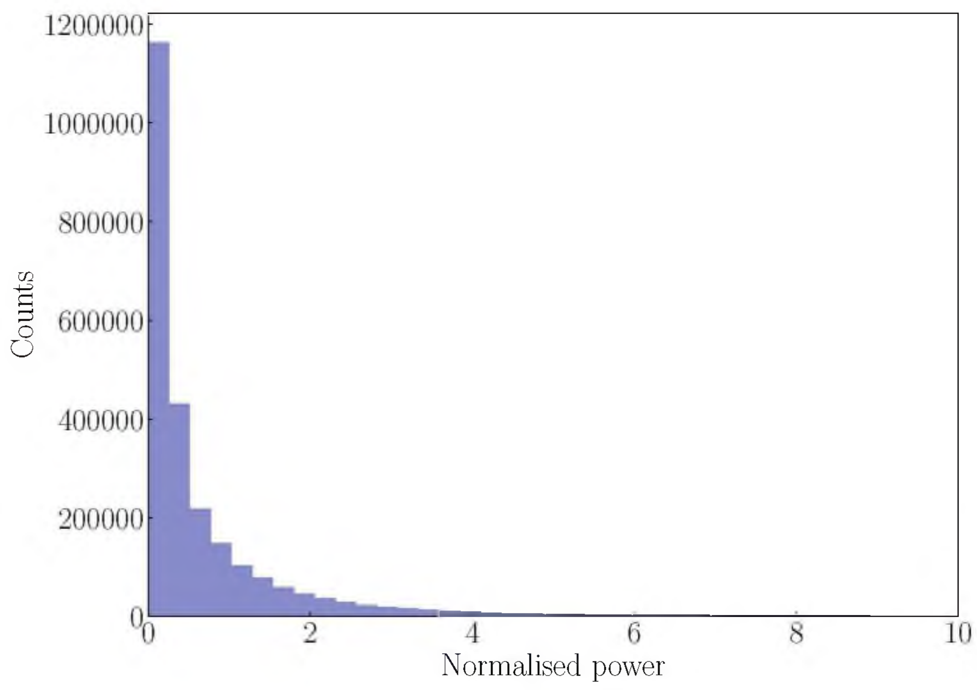


# Multi-scale Non-Gaussian Segmentation

# Multi-scale non-Gaussian segmentation (MnGSeg)

Denoising algorithm

Azzalini, A., Farge, M., & Schneider, K. 2005
Applied and Computational Harmonic Analysis, 18, 177

$$\sigma^2_{l,\theta}(\Phi) = \frac{1}{N_{l,\theta}(\Phi)} \sum_{\vec{x}} \mathbb{L}_\Phi(|\tilde{f}_{l,\theta}(\vec{x})|)|\tilde{f}_{l,\theta}(\vec{x})|^2$$

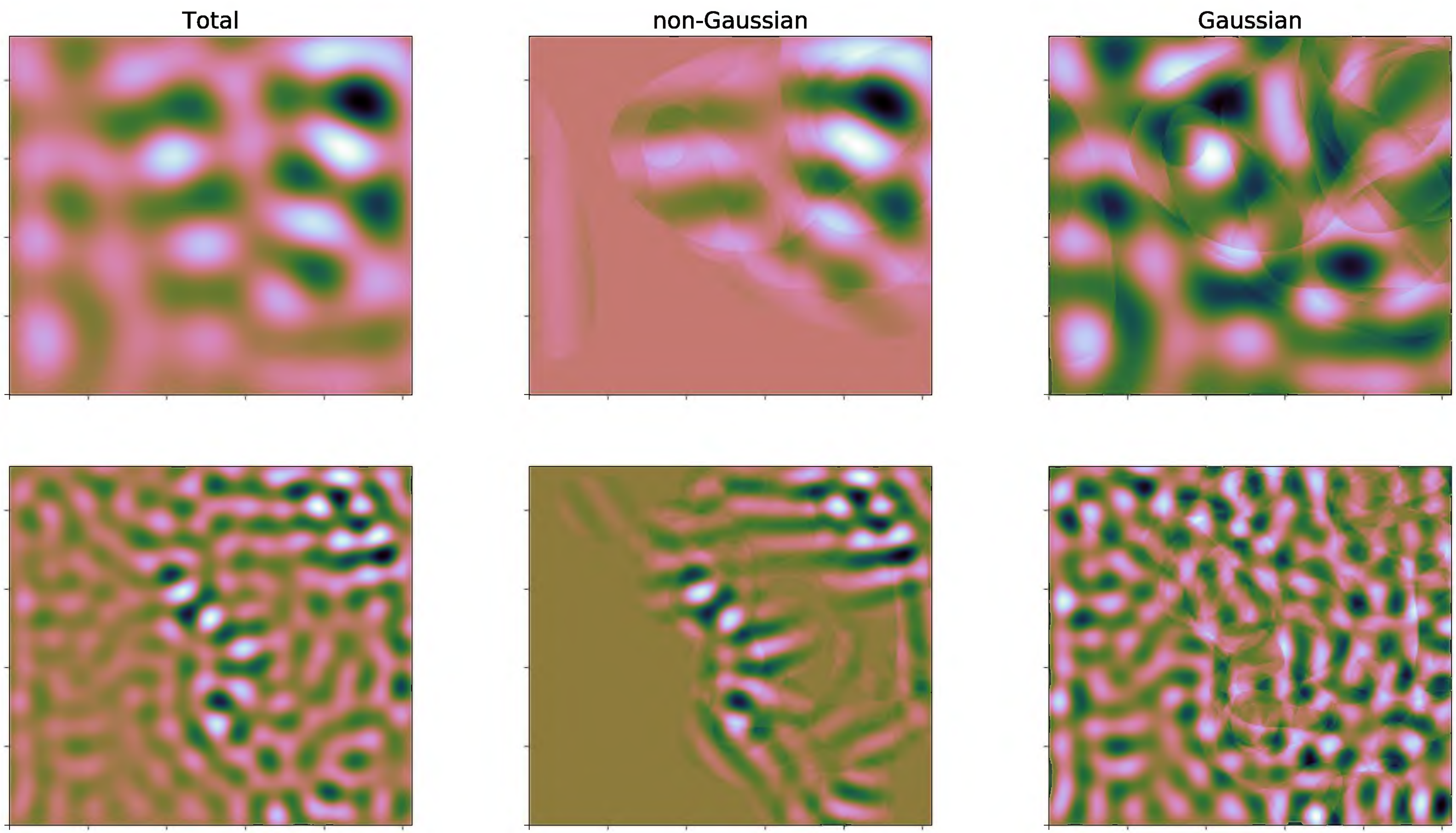
$$\mathbb{L}_\Phi(|\tilde{f}_{l,\theta}(\vec{x})|) = \begin{cases} 1 & \text{if} \quad |\tilde{f}_{l,\theta}(\vec{x})| < \Phi \\ 0 & \text{else.} \end{cases}$$

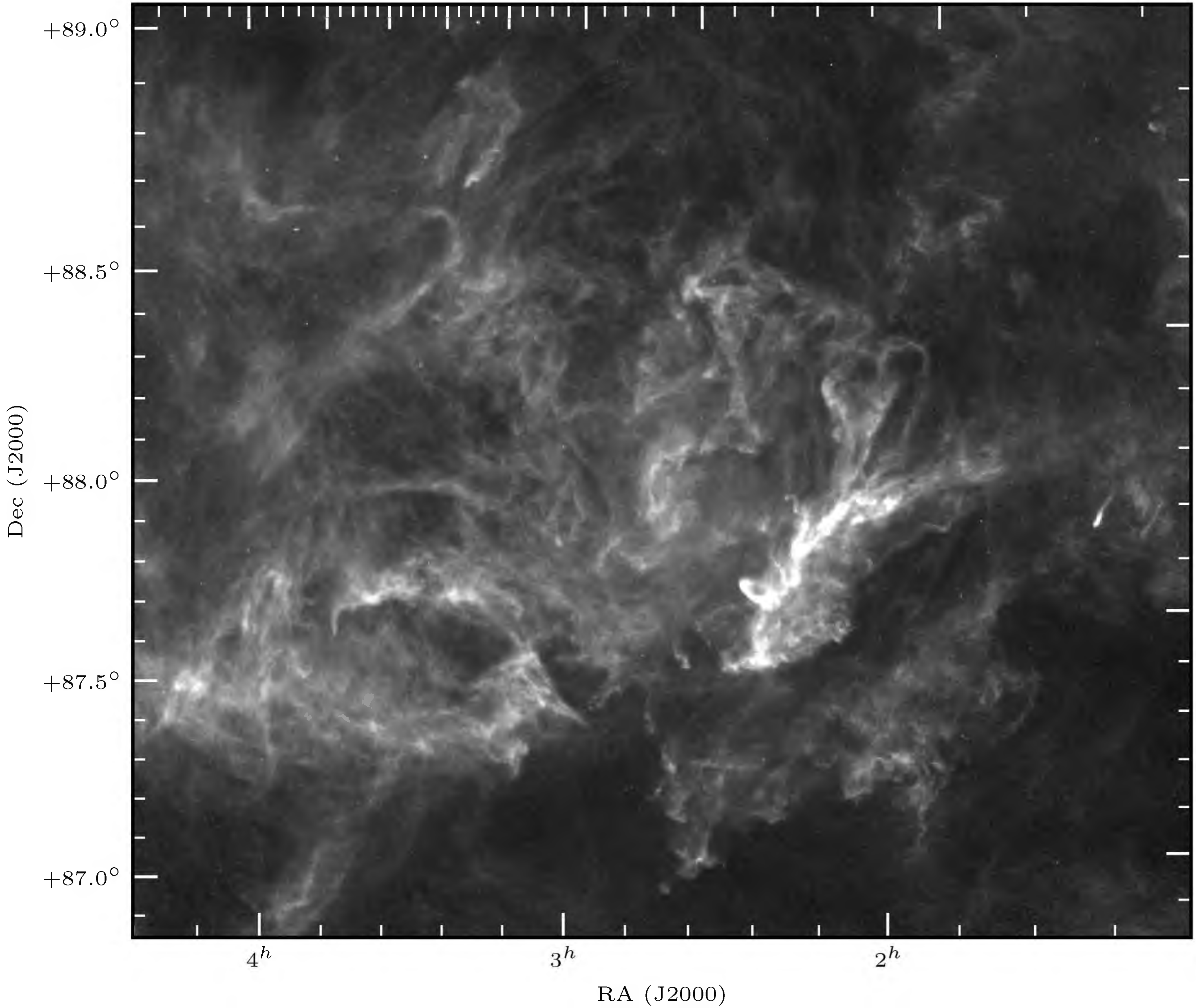
$$N_{l,\theta}(\Phi) = \sum_{\vec{x}} \mathbb{L}_\Phi(|\tilde{f}_{l,\theta}(\vec{x})|)$$

$$\begin{cases} \Phi_0(l,\theta) = \infty \\ \Phi_{n+1}(l,\theta) = q\sigma_{l,\theta}(\Phi_n(l,\theta)), \end{cases}$$

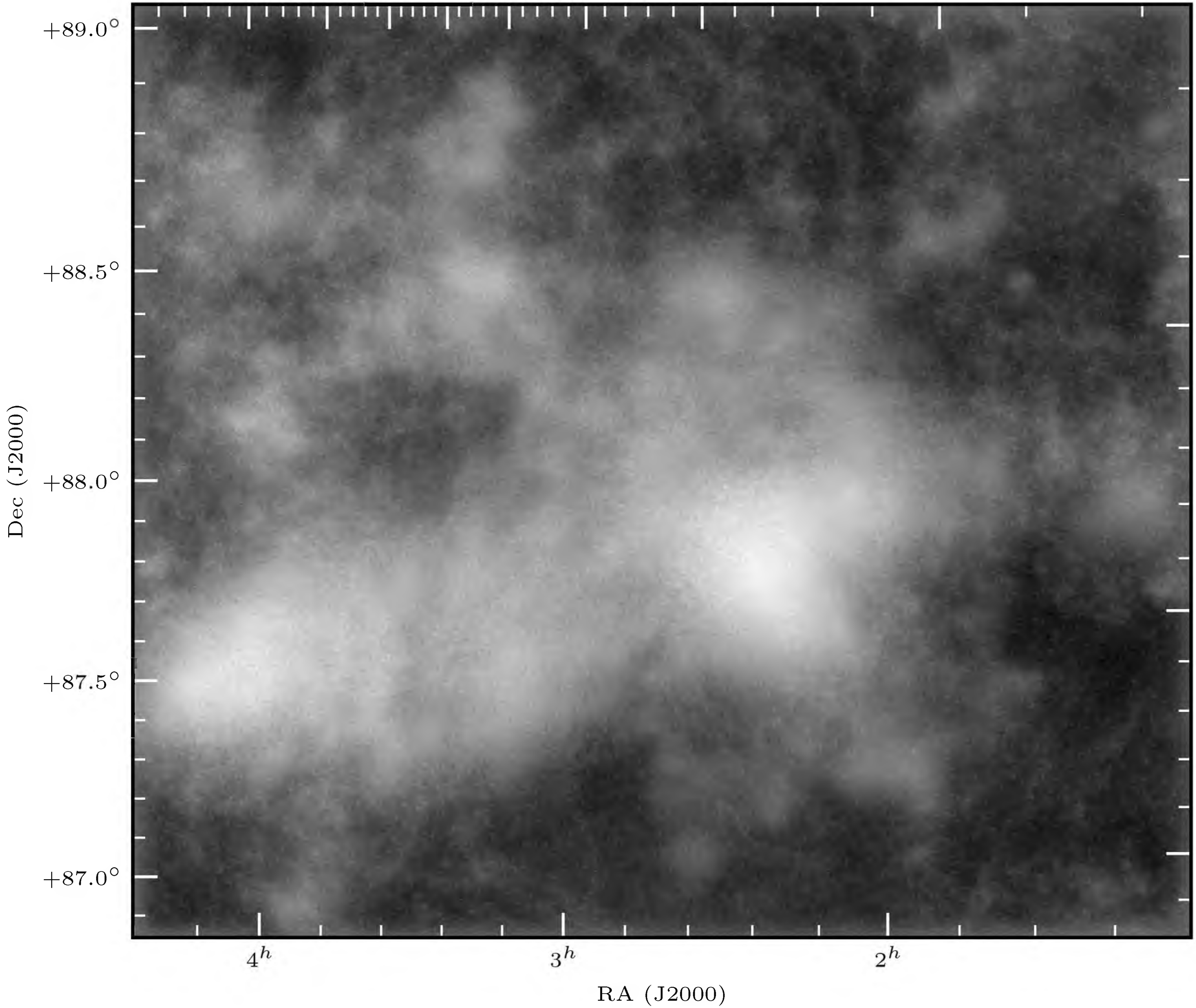
# Multi-scale non-Gaussian segmentation (MnGSeg)

Apply an iterative algorithm as a function of scales and angles.





+89.0°
+88.5°
+88.0°
+87.5°
+87.0°
Dec (J2000)
4$^h$
3$^h$
2$^h$
RA (J2000)



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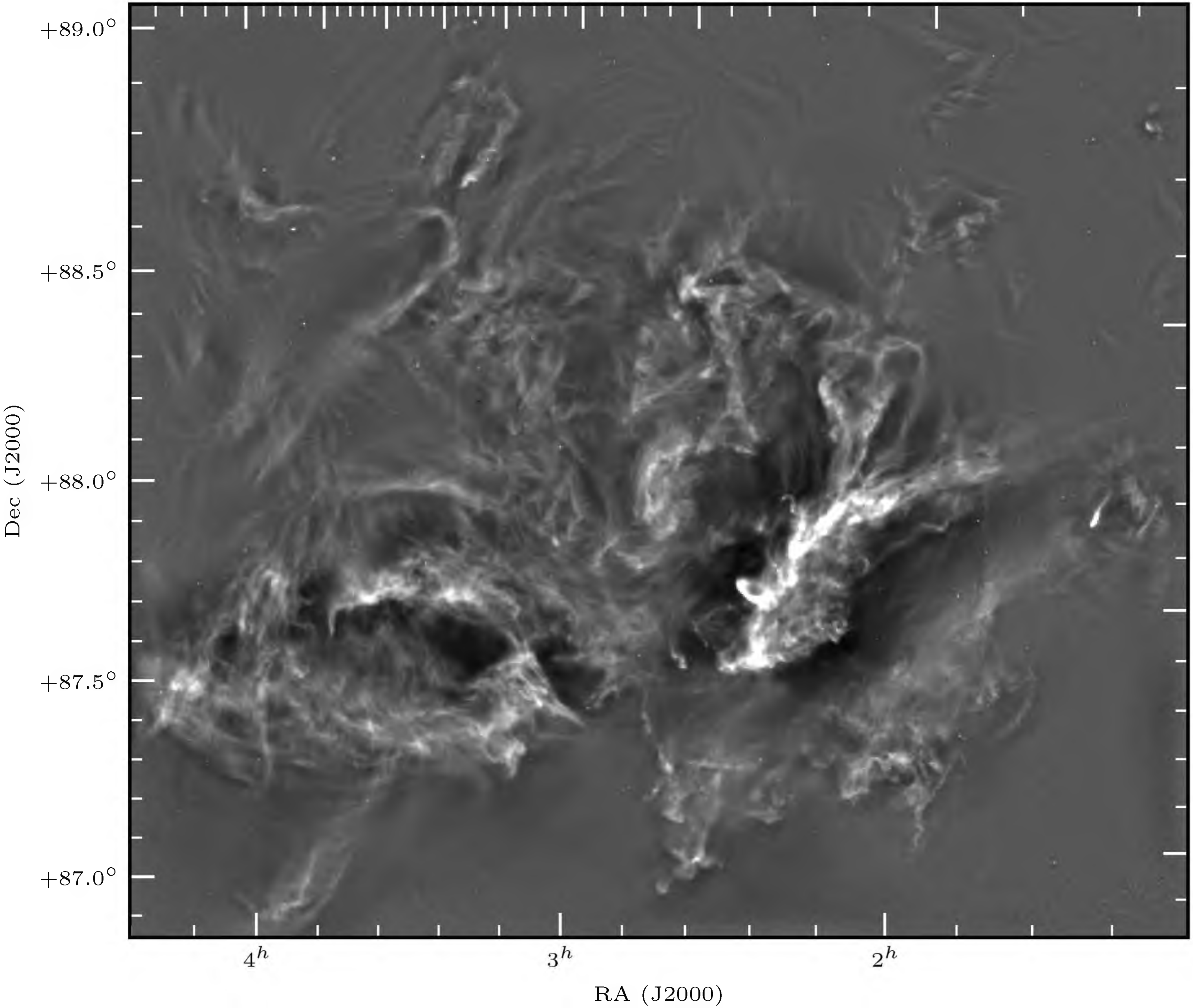

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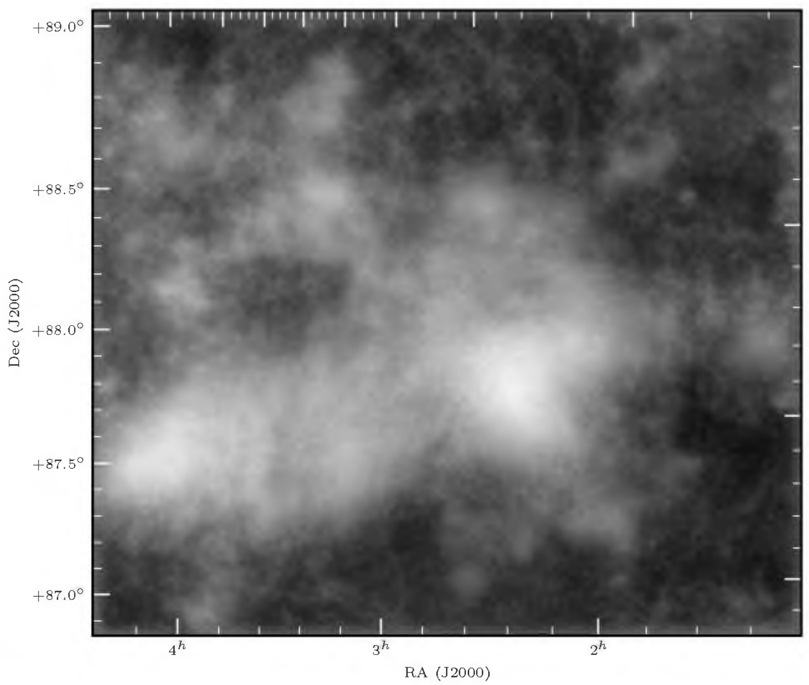
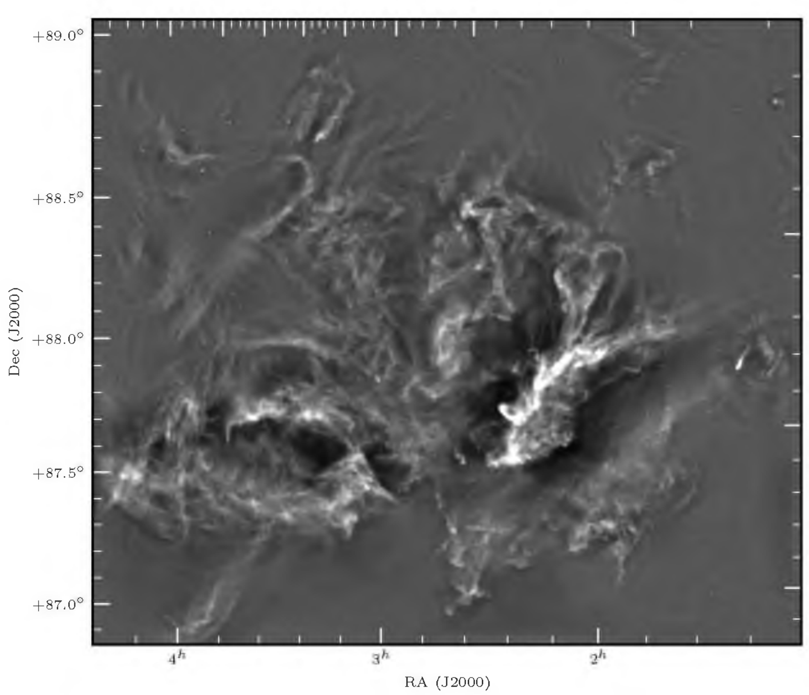
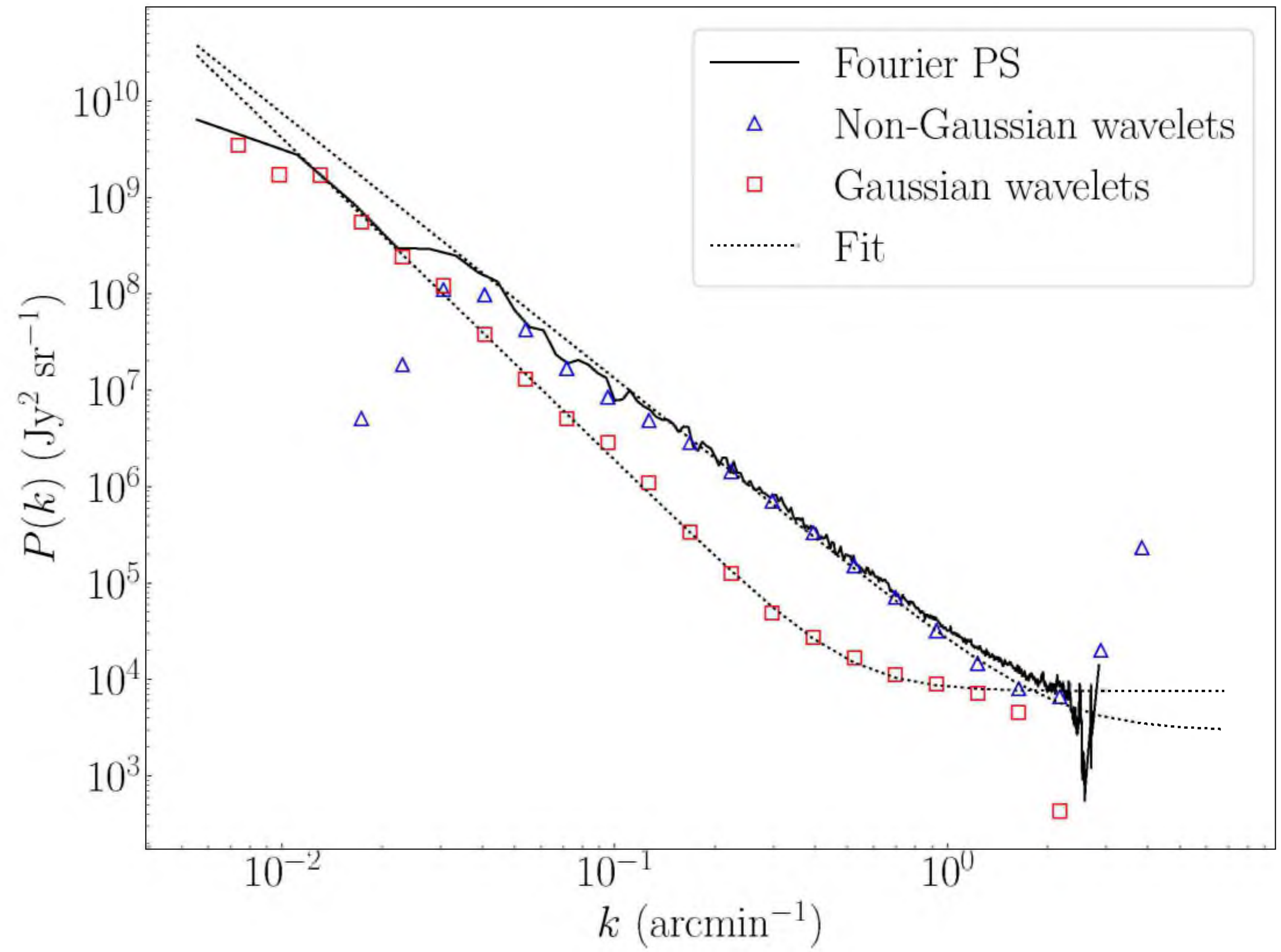
# Multi-scale non-Gaussian segmentation (MnGSeg)

Gaussian



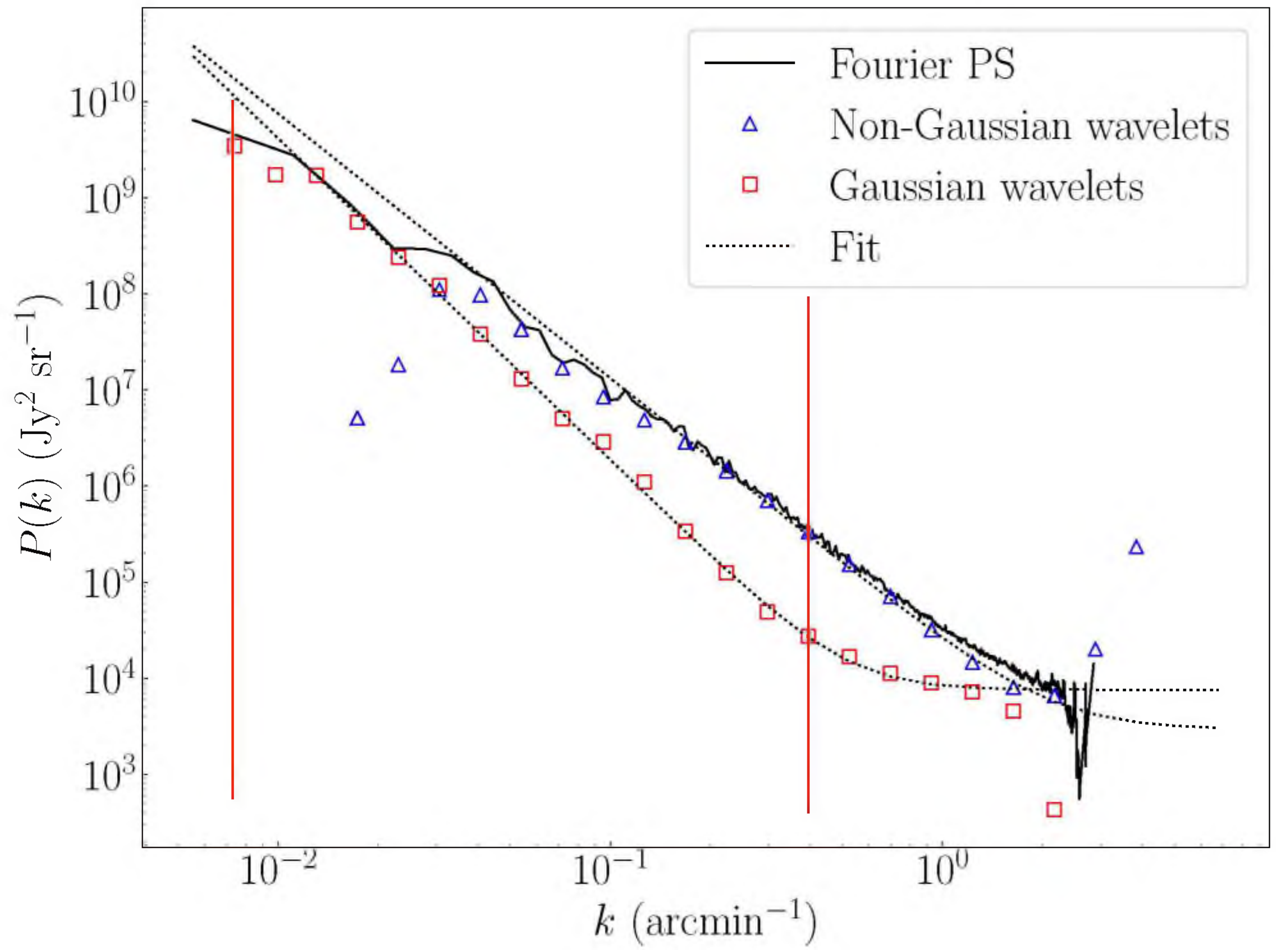

non-Gaussian

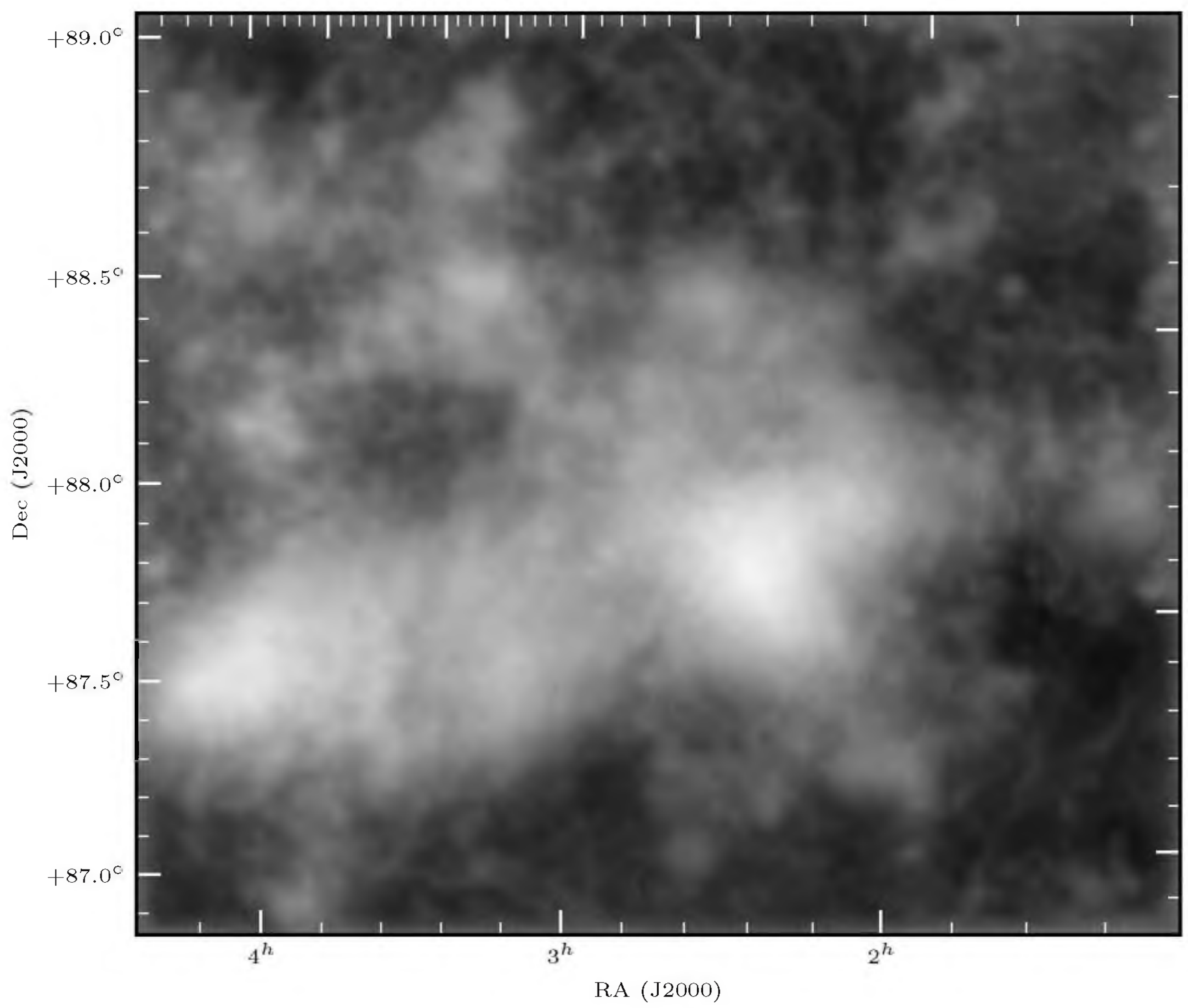



| | $A_{\rm ISM}$ (Jy$^2$ sr$^{-1}$) | Power-Law ($\gamma$) | $P_0$ (Jy$^2$ sr$^{-1}$) |
|---|---|---|---|
| Total | $(3.1 \pm 0.3) \times 10^4$ | $-2.58 \pm 0.01$ | $(3.5 \pm 0.1) \times 10^3$ |
| Gaussian | $(8 \pm 1) \times 10^2$ | $-3.34 \pm 0.05$ | $(7.6 \pm 0.7) \times 10^3$ |
| non-Gaussian | $(2.3 \pm 0.2) \times 10^4$ | $-2.75 \pm 0.07$ | $(2.9 \pm 0.8) \times 10^3$ |

# Polaris 250 µm (Gaussian part)



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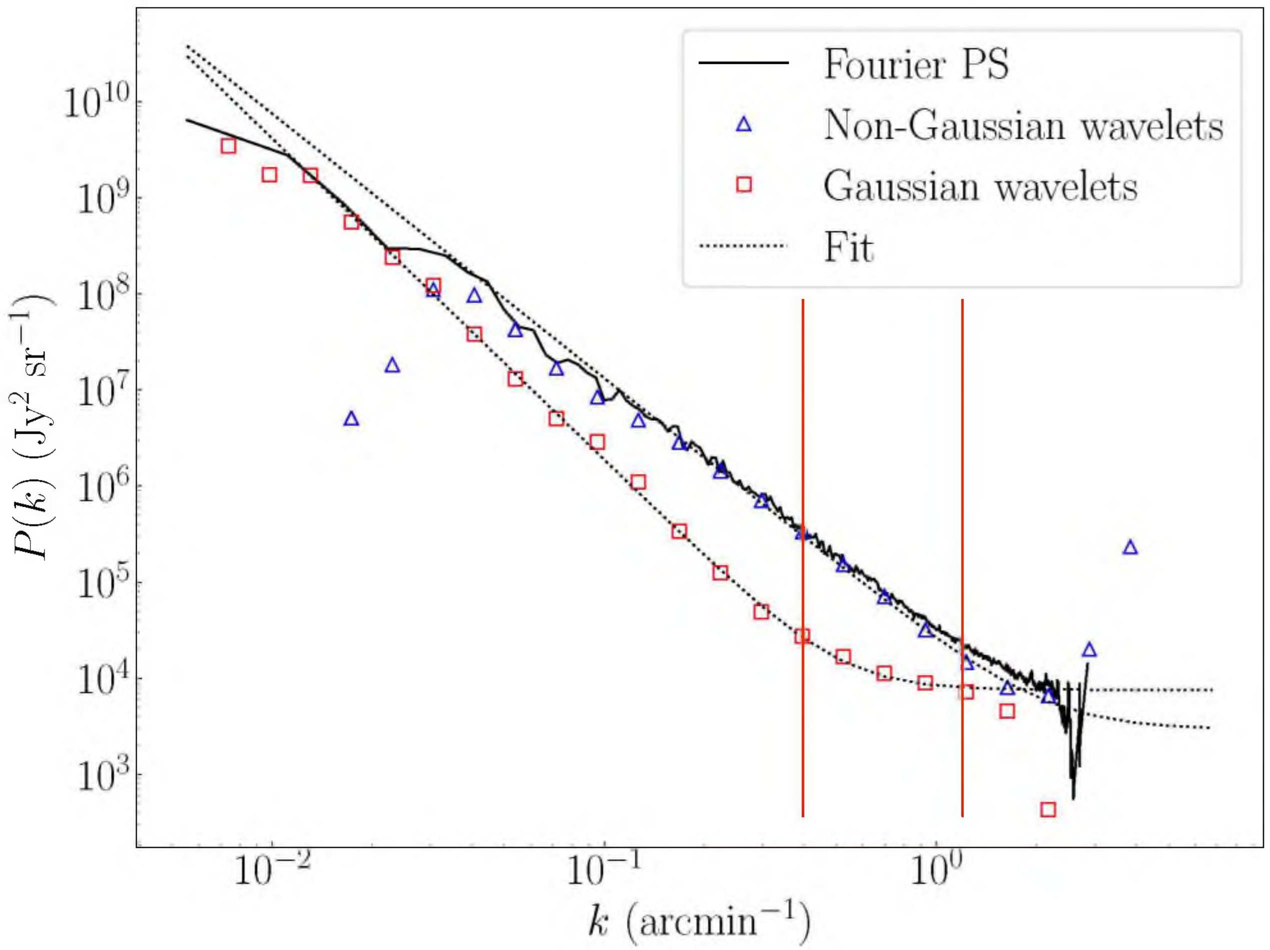
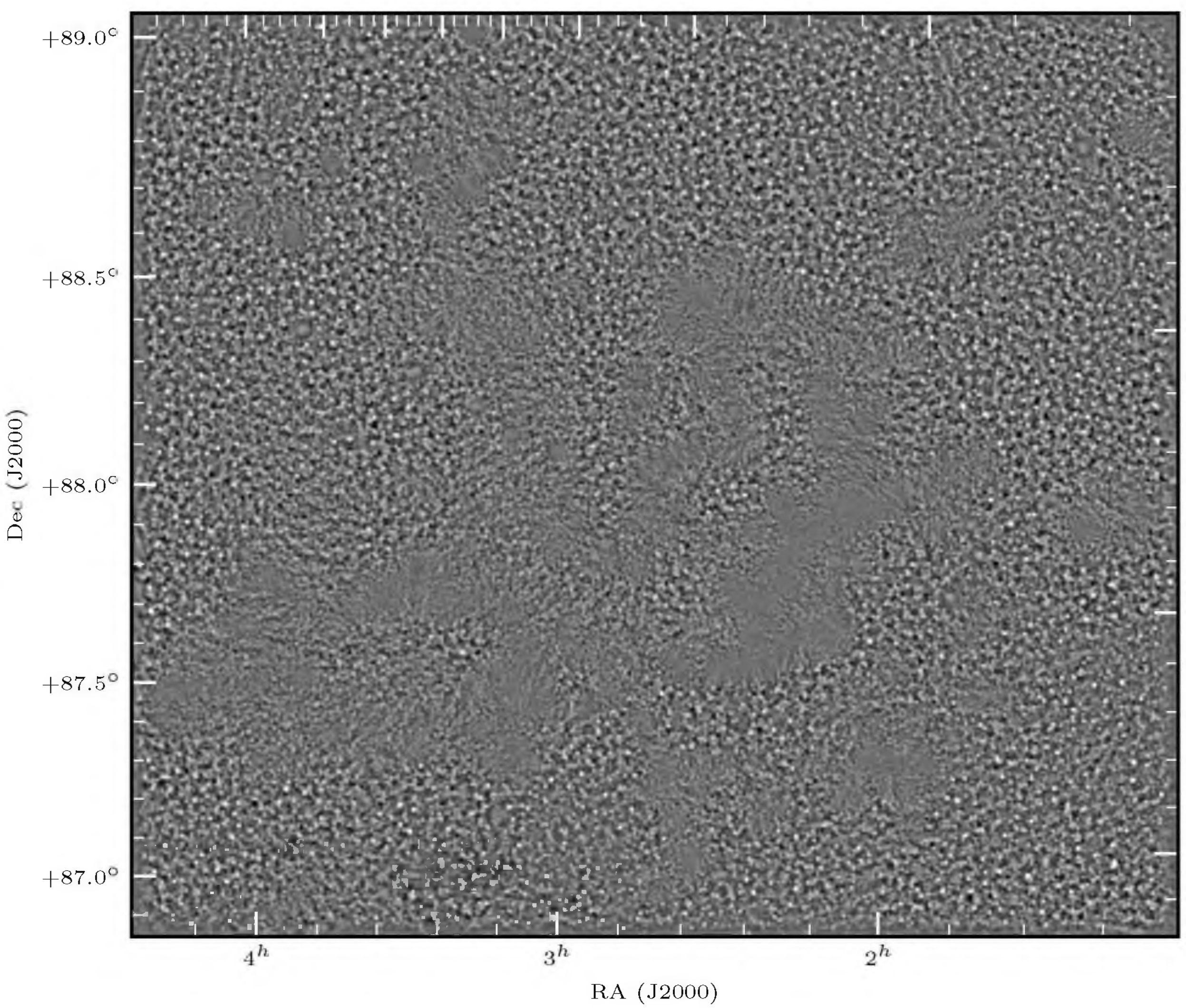
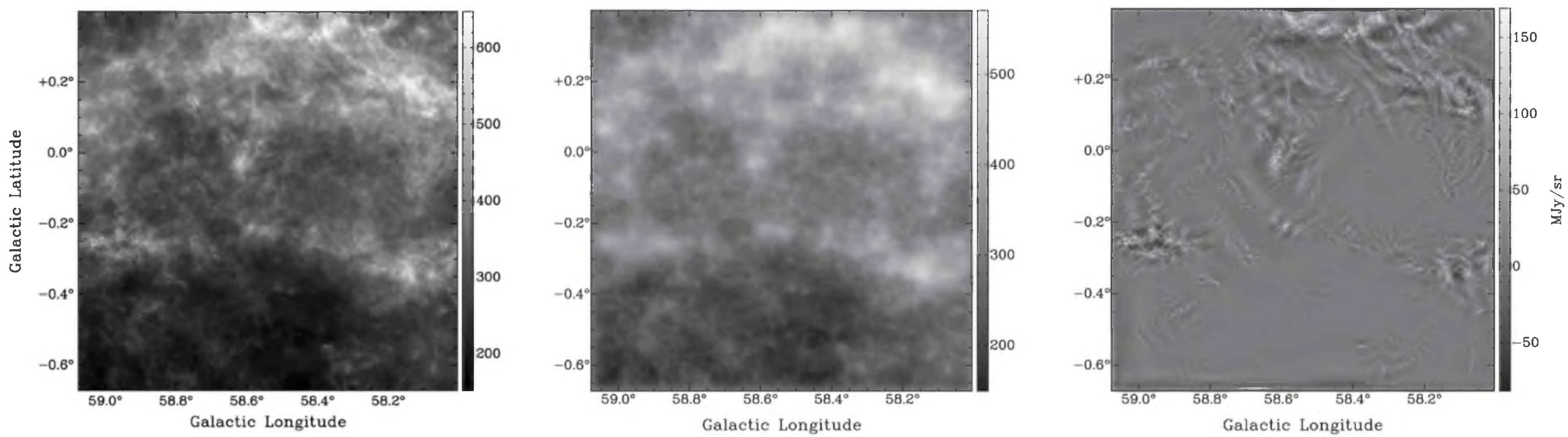



## CIB - Cosmic Infrared Background

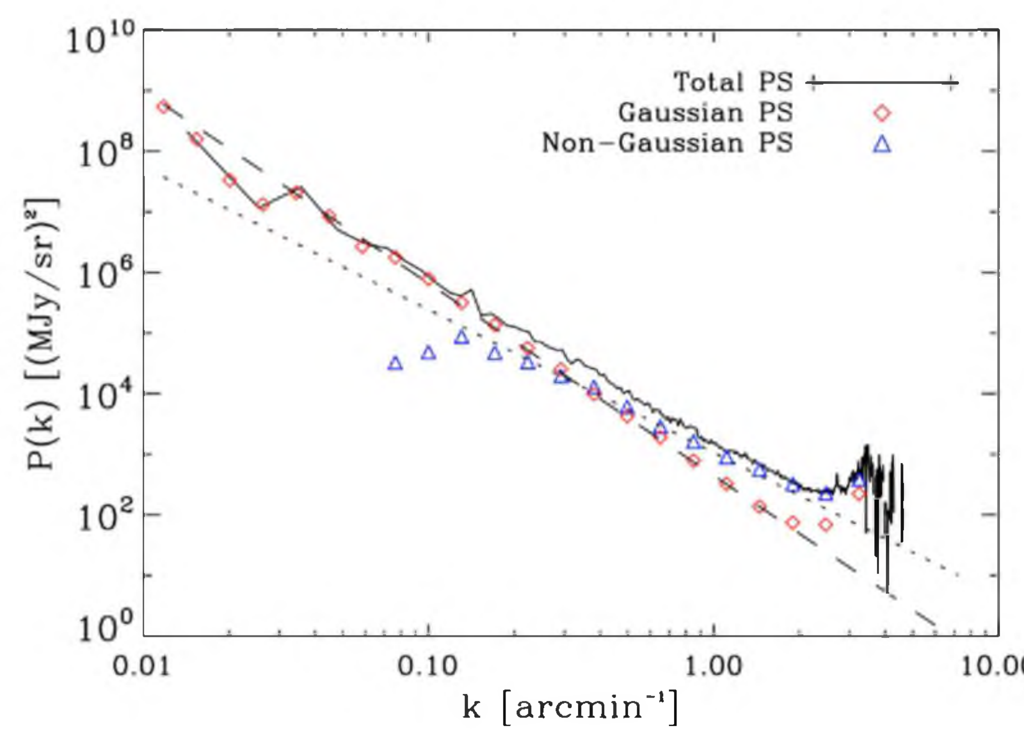


# The lack of a characteristic scale

Robitaille et al. 2014



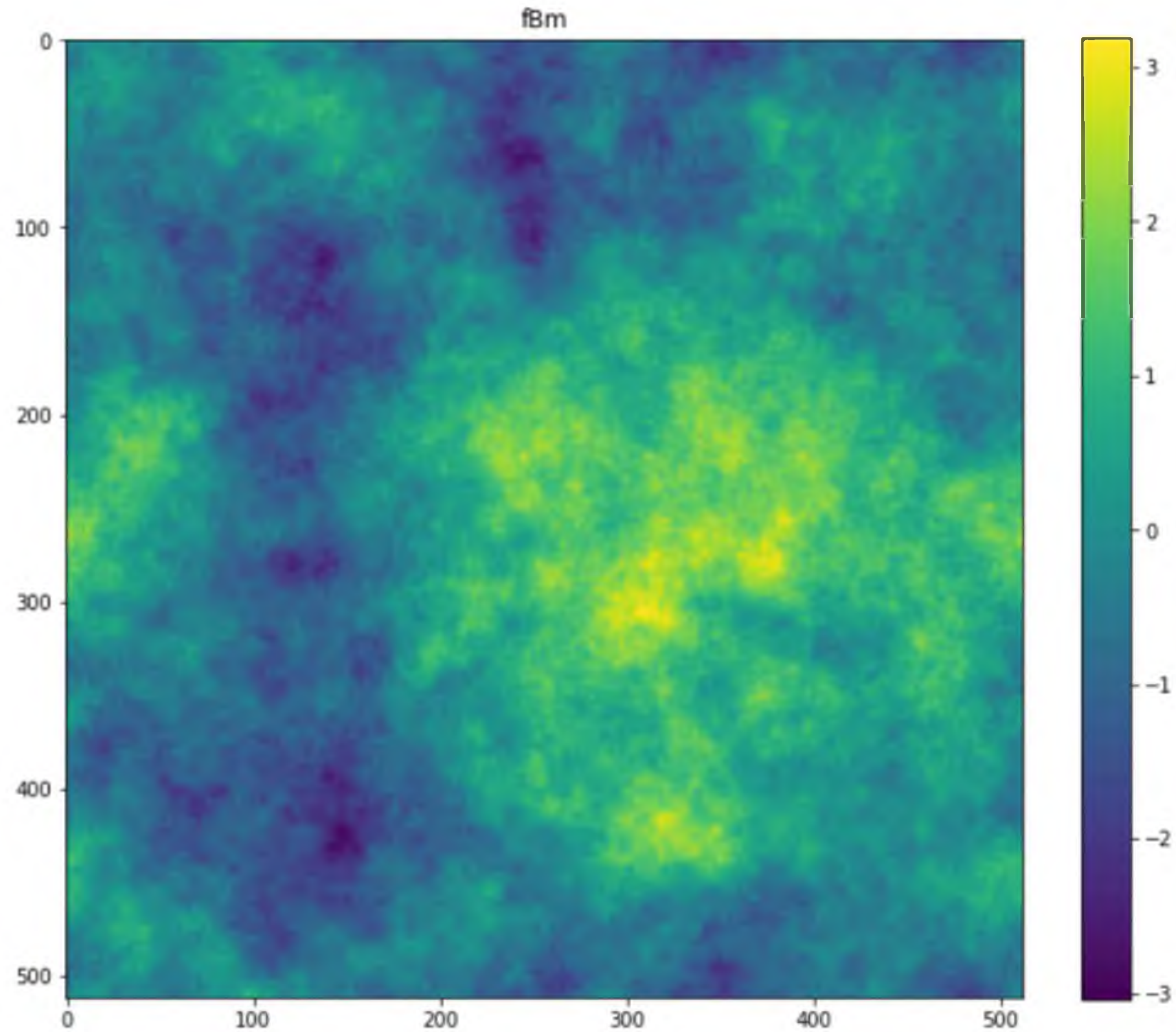

- Even when coherent structures appear at smaller scales, filaments produce no break in the power spectrum.
- Coherent structures demonstrate a spatial hierarchical scaling, i.e. filaments inside filaments, which “induces” a power law in the power spectrum.
- Filaments are well embedded in the diffuse background.



# Multiplicative random cascade model

Vázquez-Semadeni 1994

In particular, a lognormal pdf appears to be a likely candidate for the density distribution in fully turbulent flows, in which each new value of the local density is essentially random. In this case the density after a finite time can be considered to be the product of a large number of random fluctuations $[\rho(t_n) = \delta_n \delta_{n-1} \ldots \delta_1 \delta_0 \rho(t_0)]$. If these fluctuations are independent of one another, then the local density $\rho(x, t)$ should have a lognormal distribution in time.



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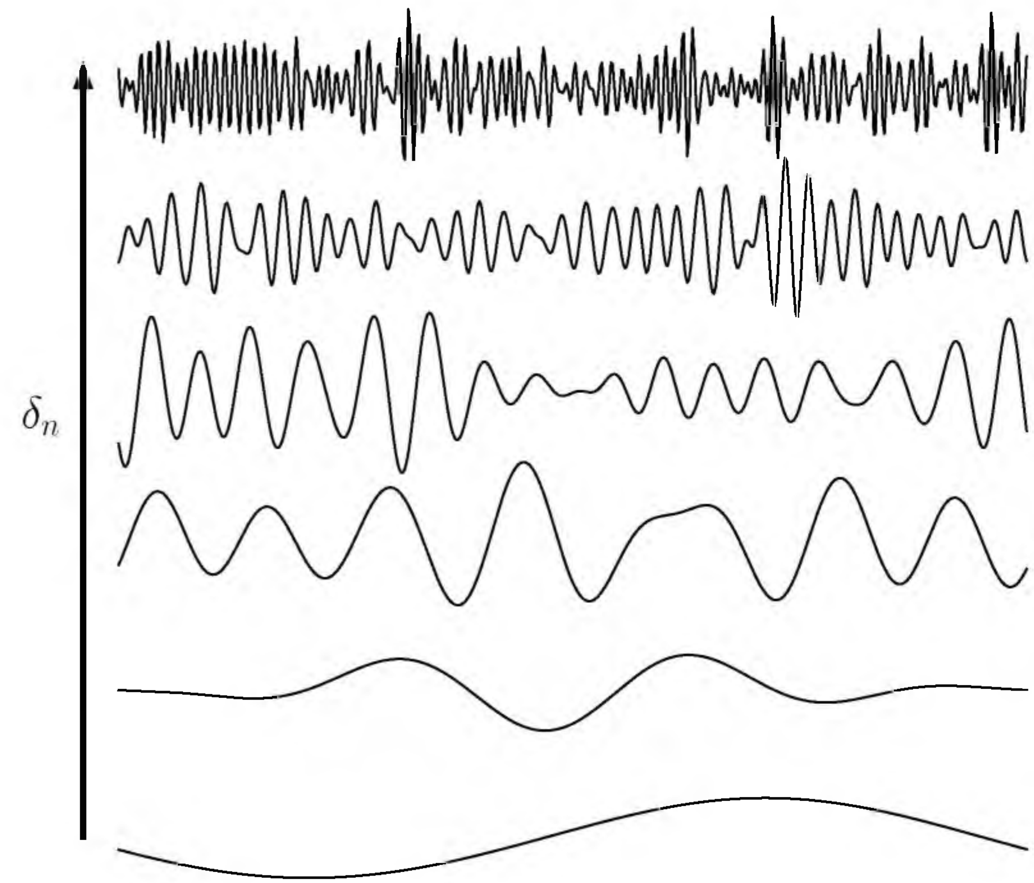


"One of the simplest methods — and historically the first — for obtaining multifractal dissipation measures" (Frisch 1995 — on intermittency models)

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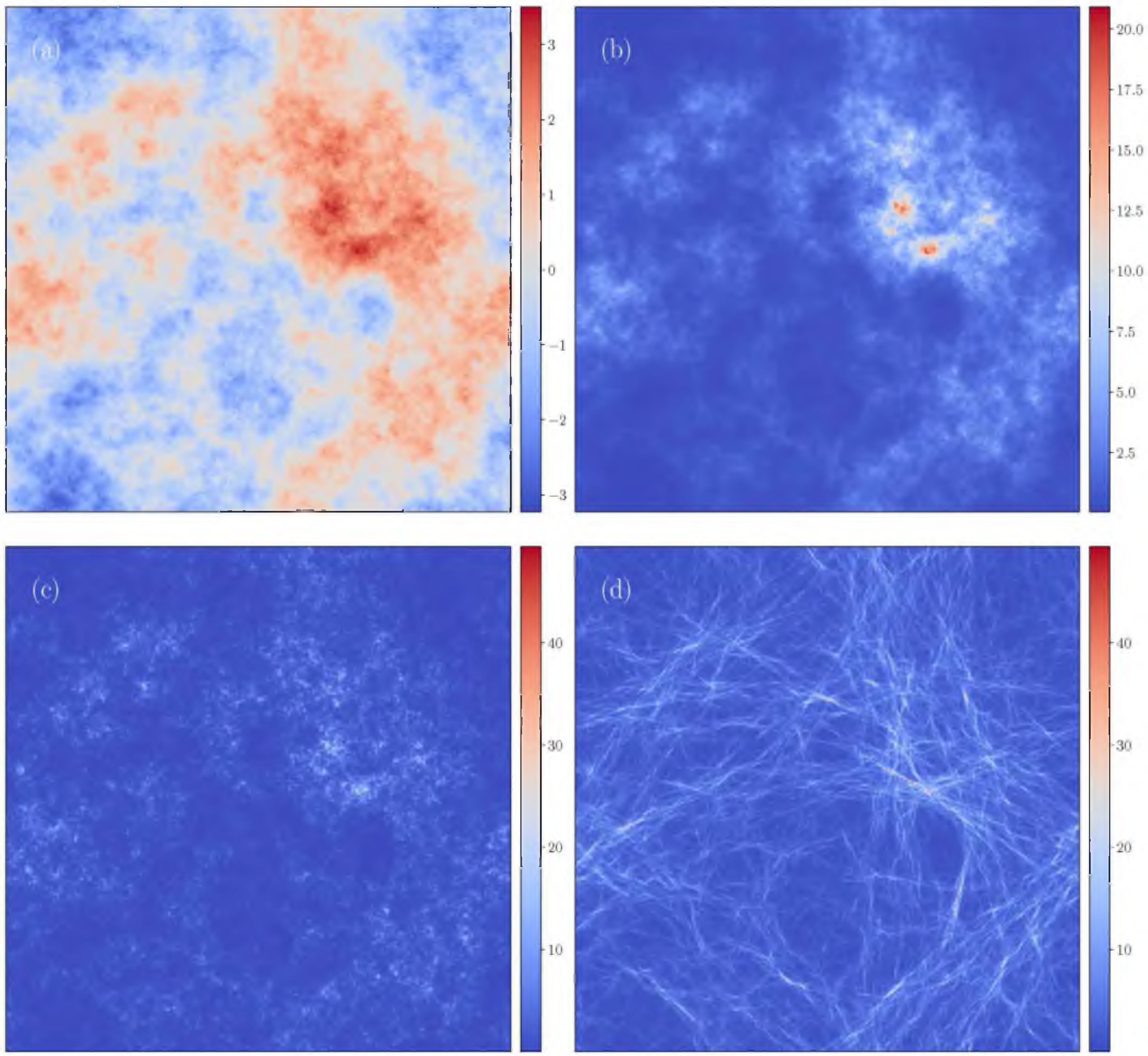


$$\rho_a(\mathbf{x}) = f(\mathbf{x})$$
$$= \textbf{fBm}$$

$$\rho_b(\mathbf{x}) = \exp[\sigma_\rho f(\mathbf{x})]$$
$$\sigma_\rho = (\ln[1 + 0.5\mathcal{M}^2])^{0.5}$$

$$\rho_c(\mathbf{x}) = \prod_{l,\theta} \exp\left[\sigma C_\ell \tilde{f}(\mathbf{x}, \ell, \theta)\right]$$

$$\rho_d(\mathbf{x}) = \frac{1}{N_\theta} \sum_\theta \left[ \prod_l \exp\left[\sigma C_\ell \tilde{f}(\mathbf{x}, \ell, \theta)\right] \right]$$

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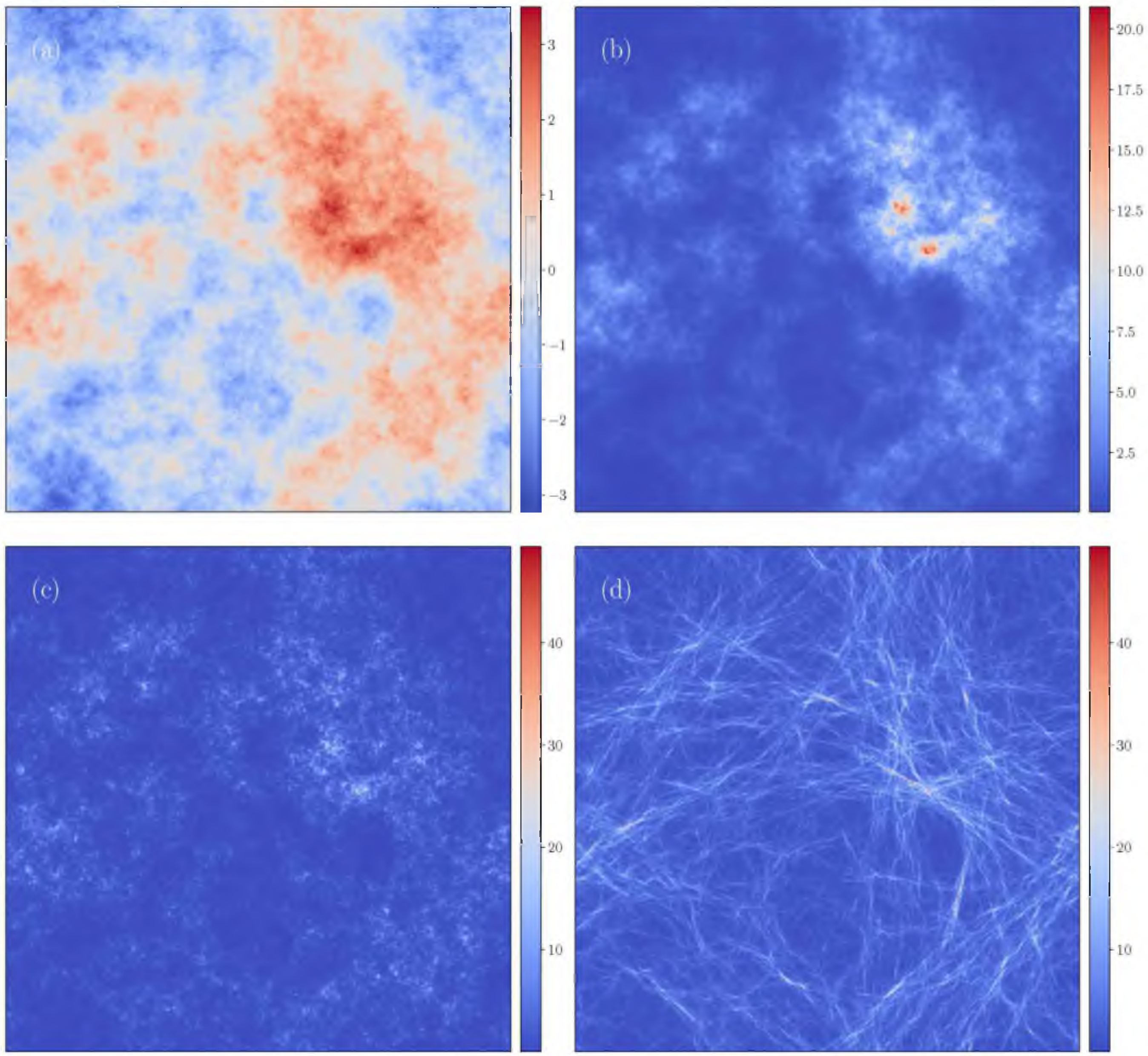


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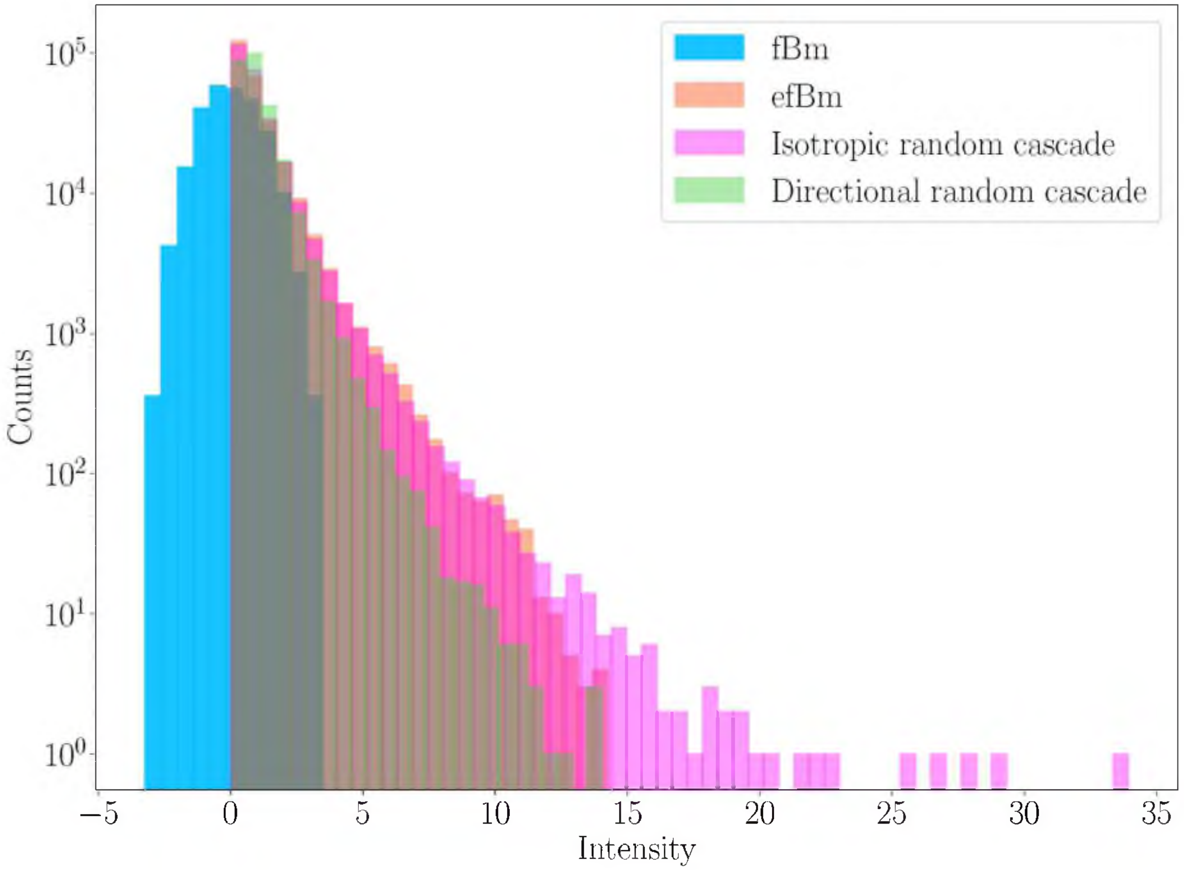
2019 (in prep.)



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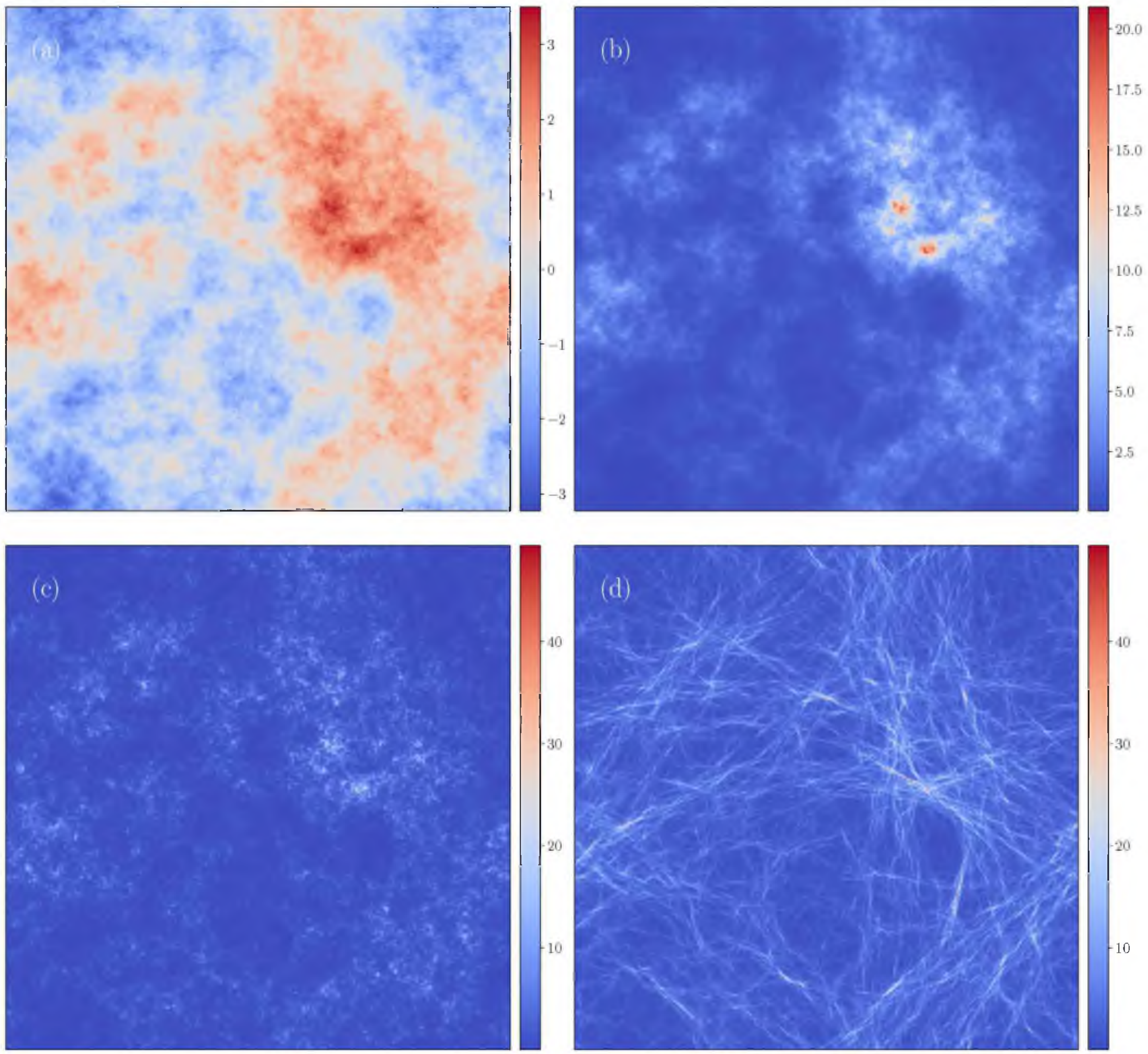


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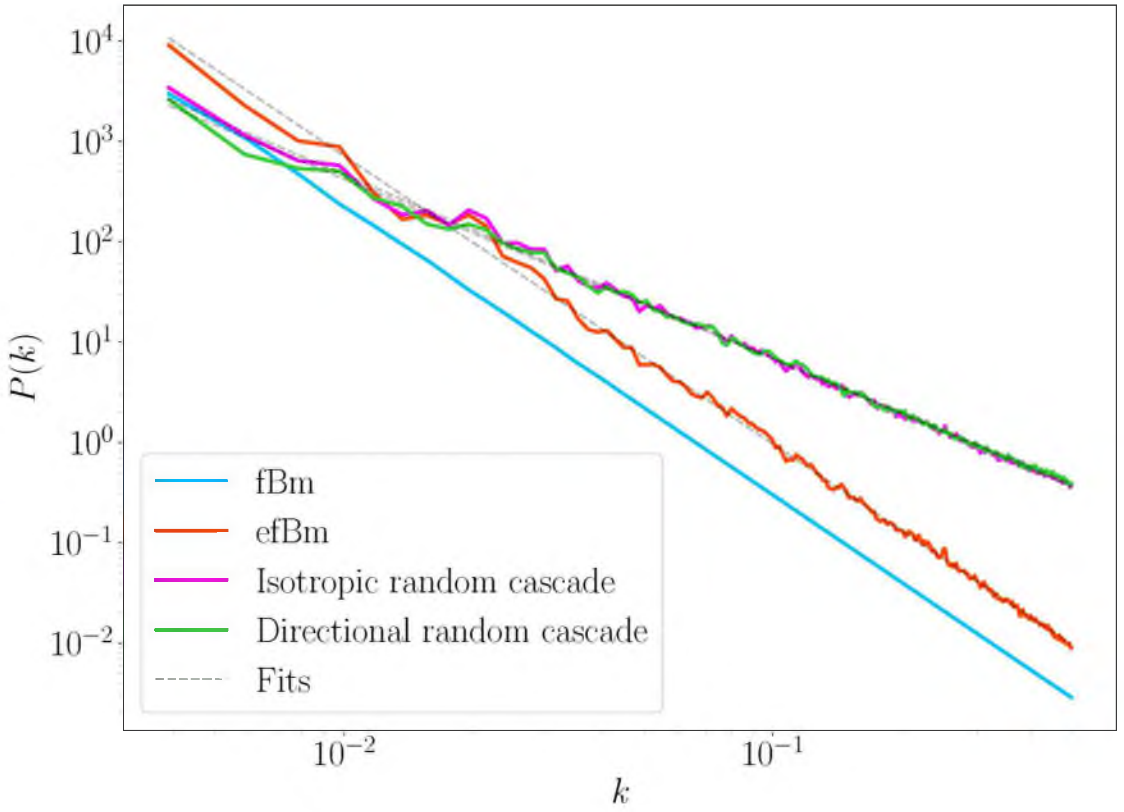

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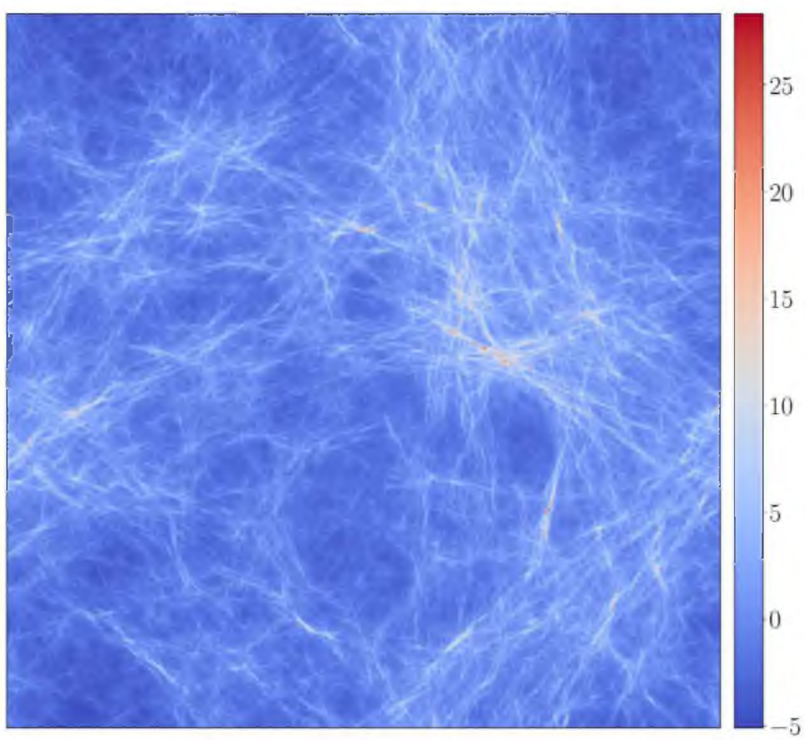


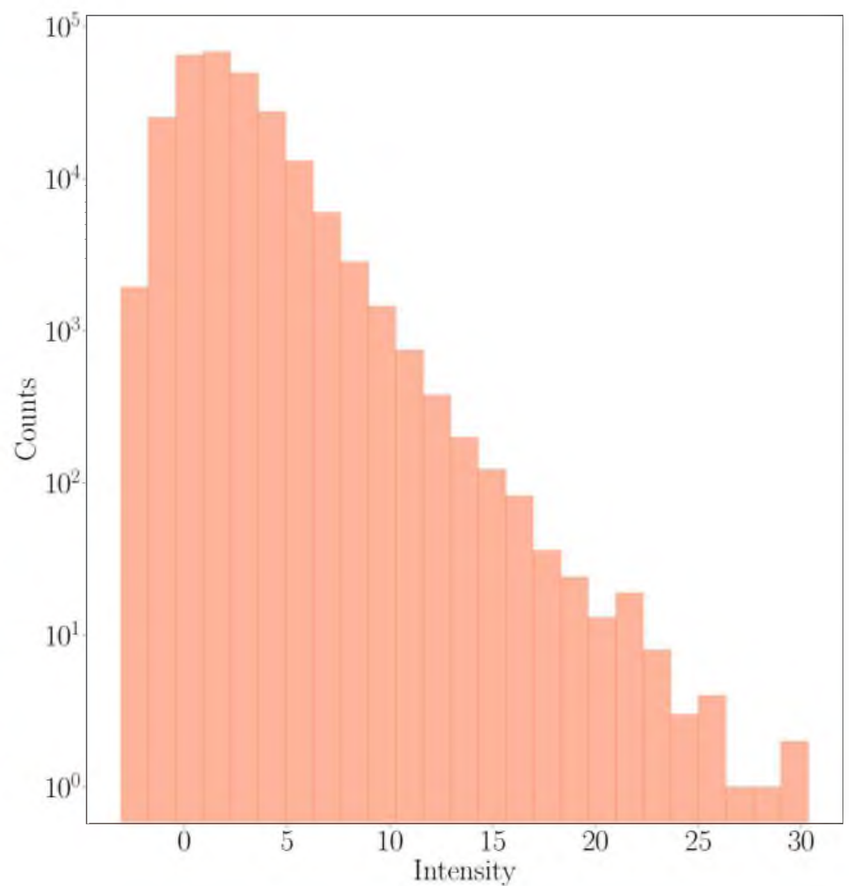


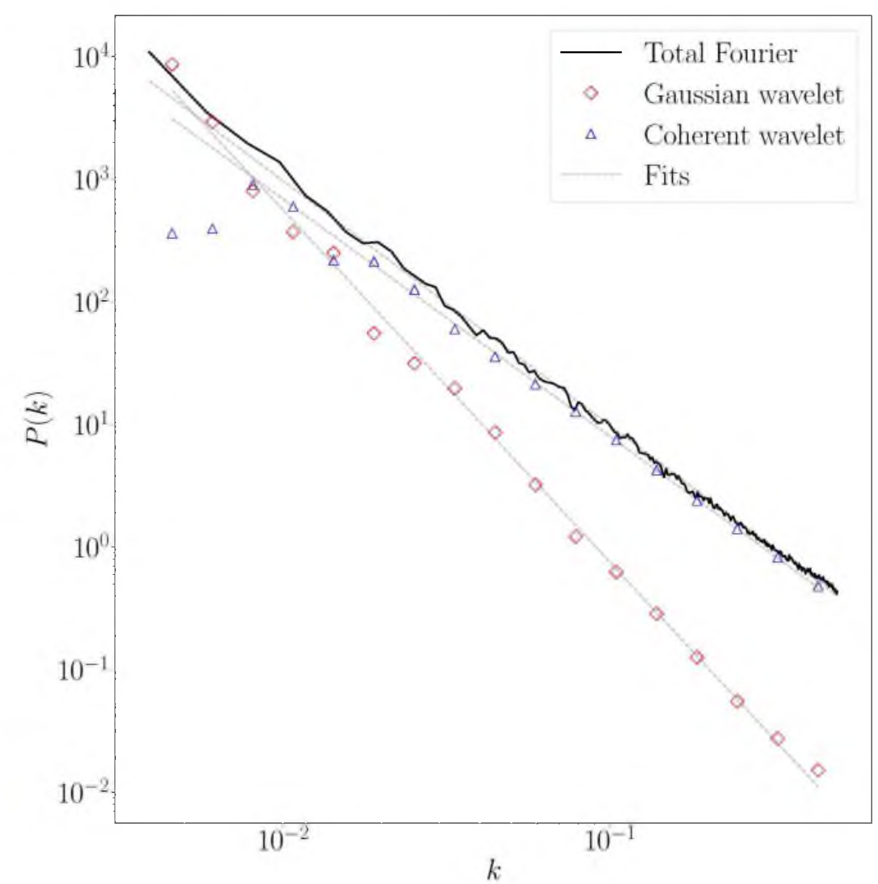


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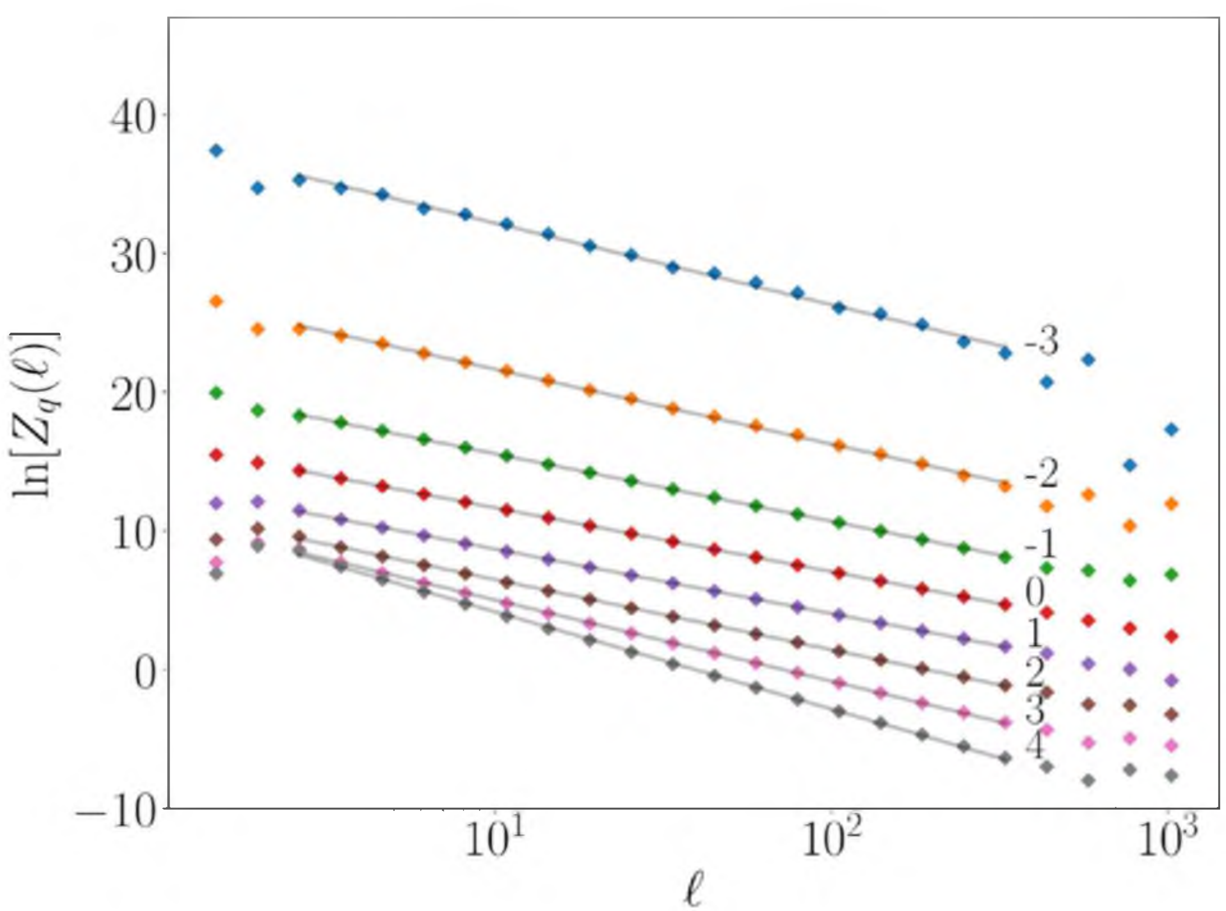
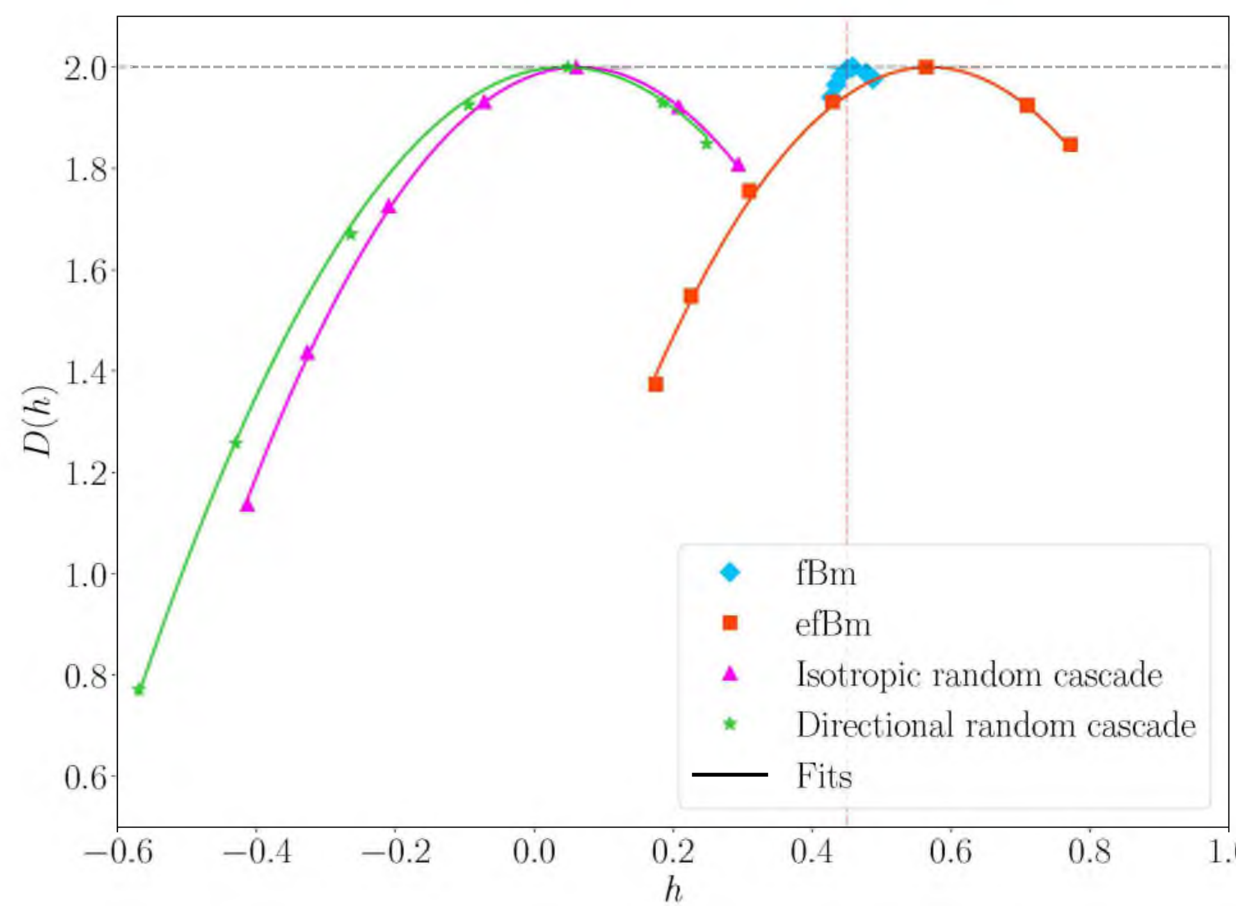
Abdelhalim Abdeldayem, M2 internship

# Multifractal analysis




Partition function

$$z(q,l) = \langle \, | \tilde{f}(l, \vec{x}, \theta) |^q \rangle_{\theta,x}$$

$$z(q,l) \sim l^{\tau(q)}$$

Multifractal spectrum

$$D(h) = qh - \tau(q)$$

$$h = \frac{d\tau(q)}{dq}$$

$$\gamma = 2 + 2h$$

# Summary

- In-depth power spectrum analysis which take into account non-Gaussianities is possible (Robitaille et al. 2019).
- This technique naturally extracts filamentary structures associated with star formation from a non-uniform background… and the CIB!
- Coherent structures associated with star formation demonstrate a spatial hierarchical scaling well embedded in the diffuse background.
- The statistical properties respect a multifractal geometry, which may be associated with turbulence intermittency.
- MnGSeg code available at https://github.com/jfrob27/pywavan